Chapter 37. Relativity

1. Newton's laws and Maxwell's equations describe the motion of charged particles and the propagation of electromagnetic waves under circumstances where the Quantum effects we discussed last week can be ignored.

2. There are some inconsistencies when the speed of motion of <u>objects</u> or <u>observers</u> approaches the speed of light.

3. These inconsistencies are resolved by Einstein's Special Theory of relativity

   The General theory describes gravitation and accelerating observers.

   The Special theory addresses modifications of Newton's Laws and relations between measurements made by different observers

Classical Physics

Maxwell's Equations

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -\frac{d}{dt}\int_{Surface} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = \mu_0 \left(I_{through} + \varepsilon_0 \frac{d}{dt}\int_{surface} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}\right)$$

Describes EM waves

$\xrightarrow{\mathrm{E,\ B}}$

$\xleftarrow{\mathrm{Q_{in},\ I_{thr}}}$

Newton's Laws

$$m\frac{d}{dt}\vec{\mathbf{v}}_{\mathbf{i}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}}_{\mathbf{i}} \times \vec{\mathbf{B}}\right)$$

$$\frac{d}{dt}\vec{\mathbf{x}}_{\mathbf{i}} = \vec{\mathbf{v}}_{\mathbf{i}}$$

$$Q_{in} = \sum_i q_i$$

$$I_{through}\vec{\mathbf{L}} = \sum_i q_i \vec{\mathbf{v}}_{\mathbf{i}}$$

Describes motion of particles

Special Relativity: Two components

1. How are the laws of physics modified when objects move close to the speed of light?

2. What do observers who are moving relative to each other measure when something happens? How are the measurements related?

You will be surprised to learn that very little changes in terms of the mathematical statement of the laws of physics.

You will be puzzled by the counterintuitive relations between measurements made by moving observers. Most of the conceptual difficulty is here.

Maxwell's Equations describe the excitation of electromagnetic fields by moving charges.

If charges' positions and velocities are known MEs tell us what are the electromagnetic fields, including the generation and propagation of light waves.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -\frac{d}{dt} \int_{Surface} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = \mu_0 (I_{through} + \varepsilon_0 \frac{d}{dt} \int_{surface} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}})$$

How many, and which ones need to be modified?

1. All
2. Some - first two
3. Some - second two
4. None
5. None of the above

Newton's Laws with the Lorenz force tells us how charged particles move in electromagnetic fields.

Newton's Laws

#1 $$\frac{d}{dt}\vec{\mathbf{p}}_{\mathbf{i}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}}_{\mathbf{i}} \times \vec{\mathbf{B}}\right)$$

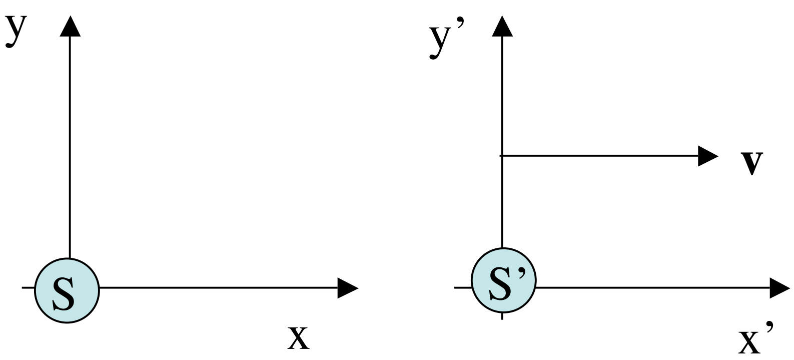
#2 $$\vec{\mathbf{p}}_{\mathbf{i}} = m\vec{\mathbf{v}}_{\mathbf{i}}$$

#3 $$\frac{d}{dt}\vec{\mathbf{x}}_{\mathbf{i}} = \vec{\mathbf{v}}_{\mathbf{i}}$$

How many, and which ones need to be modified?

1. All
2. #1
3. #2
4. #3
5. None
6. None of the above

Reference Frames: Two observers moving relative to each other measure different values for some quantities.



Observer stationary in S

Observer stationary in S’

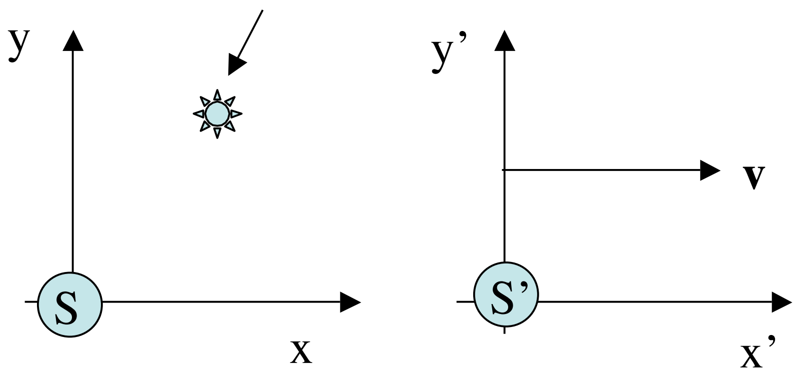
Reference frame S’ is moving at velocity v in the x direction with respect to Reference frame S.

Reference frame S is moving at velocity -v in the x direction with respect to Reference frame S’.

Inertial frames: reference frames moving at constant velocities with respect to each other, and in which the laws of physics apply.

Reference Frames: Two observers moving relative to each other measure different values for positions over time.

A light flashes



Observer stationary in S

Observer stationary in S'

Question: A light flashes. Observer S say's it flashed at time t, at the point x, y, z. When and where does Observer S' say it flashed. Assume you know nothing about relativity.

Coordinates and conventions.

1. For simplicity, align axes of reference frames so that relative motion of the frames is in one coordinate's direction, say - x.
2. Pick the origin of both systems to coincide at time t=0.

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

An object moving with x=vt in S appears stationary in S'

Galilean Transformation

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

Only difference is in coordinate in which motion occurs.

Both observers measure the same time.

Inverse transformation (v becomes -v)

$$x = x' + vt'$$

$$y = y'$$

$$z = z'$$

$$t = t'$$

Galilean Transformation addition of velocities

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

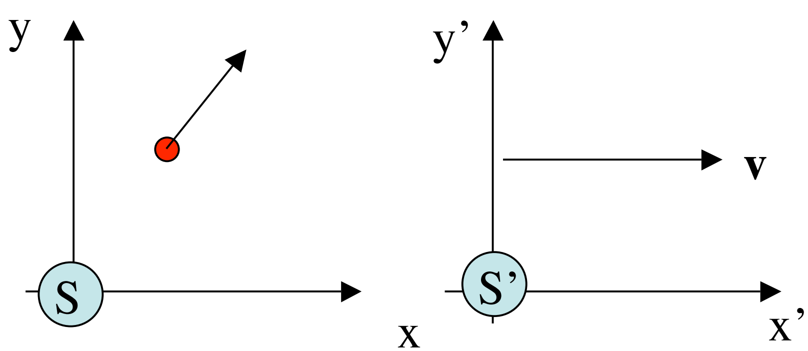
$$t' = t$$

In frame S particle is observed to move from point x_1, y_1, z_1, at time t_1 to point x_2, y_2, z_2 at time t_2

Component of velocity in x direction

$$u_x = \frac{x_2 - x_1}{t_2 - t_1}$$

Particle with velocity **u**



Velocity observed in frame S'

$$u'_x = \frac{x'_2 - x'_1}{t'_2 - t'_1}$$

$$u'_x = \frac{(x_2 - vt_2) - (x_1 - vt_1)}{t_2 - t_1}$$

$$u'_x = \frac{(x_2 - x_1) - v(t_2 - t_1)}{t_2 - t_1}$$

$$u'_x = u_x - v$$

Other components of velocity unchanged

$$u'_y = u_y$$

$$u'_z = u_z$$

For Galilean Transformations - Acceleration is invariant

Suppose the velocity measured in frame S is **u**(t).

The velocity measured in S' is **u**'(t)=**u**(t) -**v**

What is acceleration in each frame?

$$\vec{\mathbf{a}}'(t') = \frac{d}{dt'}\vec{\mathbf{u}}'(t') = \frac{d}{dt}(\vec{\mathbf{u}}(t) - \vec{\mathbf{v}}) = \frac{d}{dt}\vec{\mathbf{u}}(t) = \vec{\mathbf{a}}(t)$$

$$\vec{\mathbf{a}}'(t') = \vec{\mathbf{a}}(t)$$

So, assuming $m' = m$ and if $\vec{\mathbf{F}}' = \vec{\mathbf{F}}$

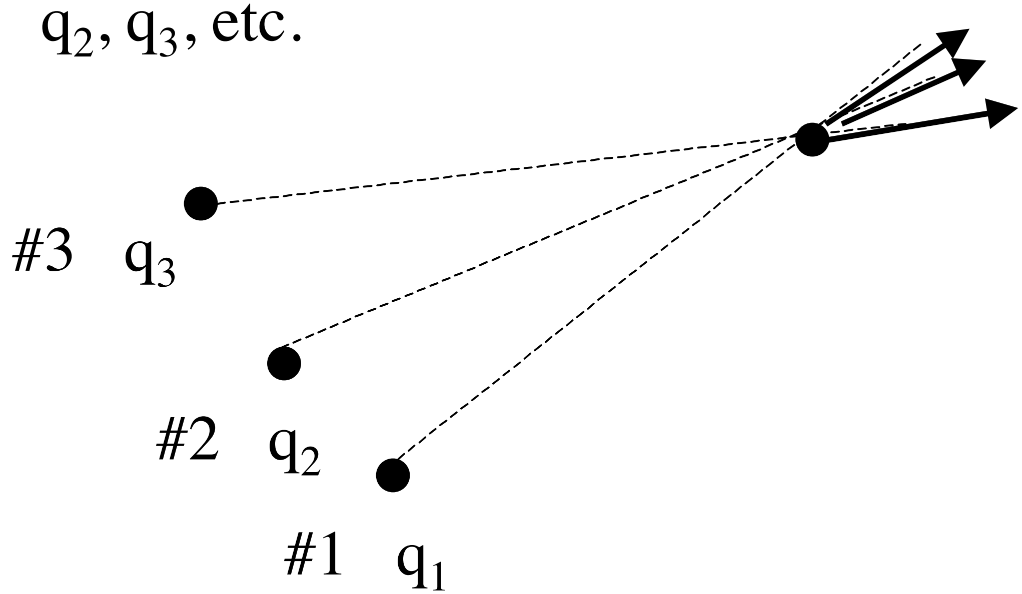
$$m\vec{\mathbf{a}}' = \mathbf{F}' \qquad m\vec{\mathbf{a}} = \mathbf{F}$$

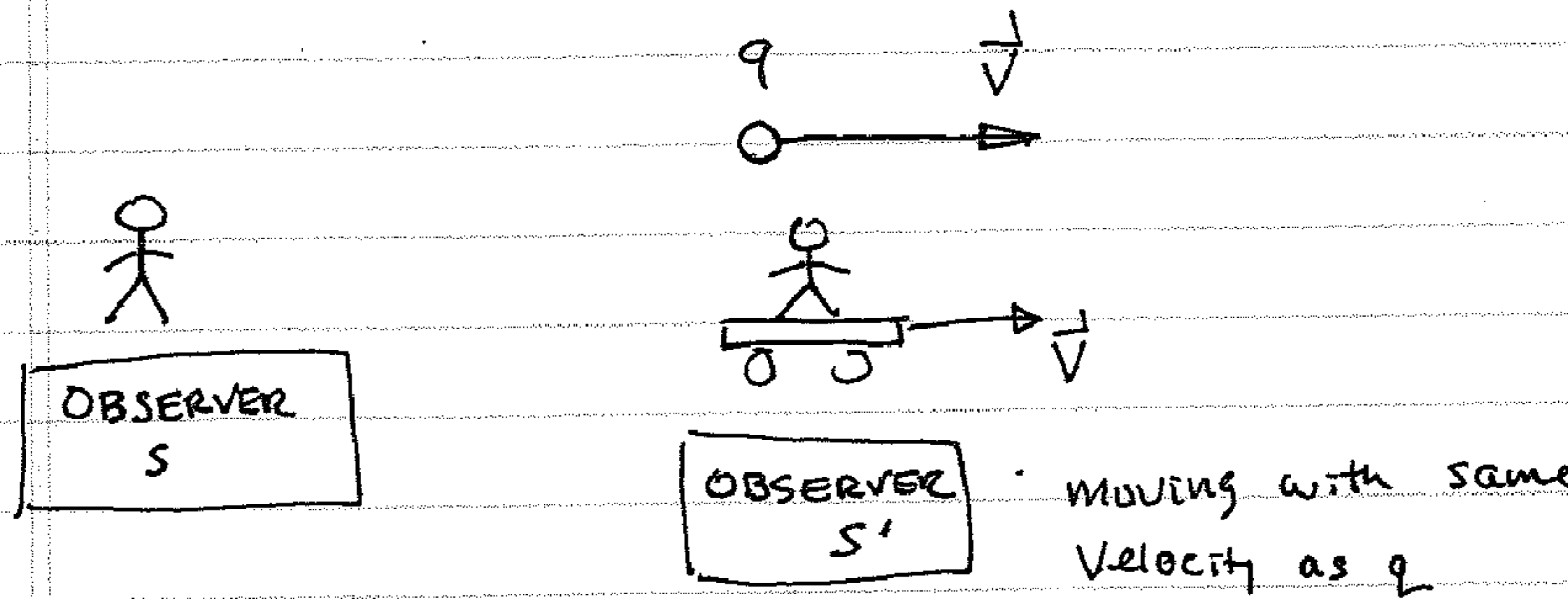
Newton's law has the same from in both frames

Suppose the force were given by Coulomb's law. Would that have the same values in all frames?

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}(\vec{\mathbf{r}}) \qquad \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \sum_{q_j} \frac{Kq_j}{r_j^2}\hat{\mathbf{r}}_{\mathbf{j}}$$

Charges making force: q_1, q_2, q_3, etc.





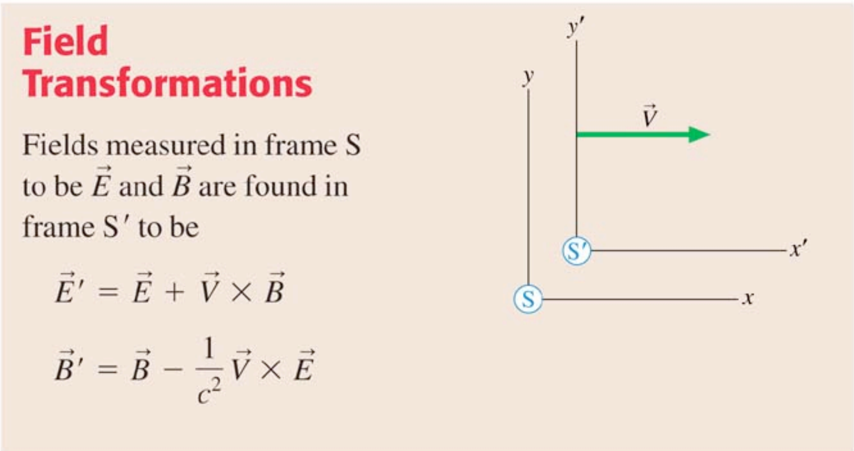
Observer S says: $\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$ q makes E and B

Observer S' says: $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$ v=0 for him, q makes E

How can both be right?

<u>Option A:</u> There is a preferred reference frame (for example S). The laws only apply in the preferred frame. But, which frame?

<u>Option B:</u> No frame is preferred. The Laws apply in all frames. The electric and magnetic fields have different values for different observers.



<u>Extended Option B:</u> No frame is preferred. The Laws apply in all frames. All observers agree that light travels with speed c. Einstein's postulates →Special Relativity

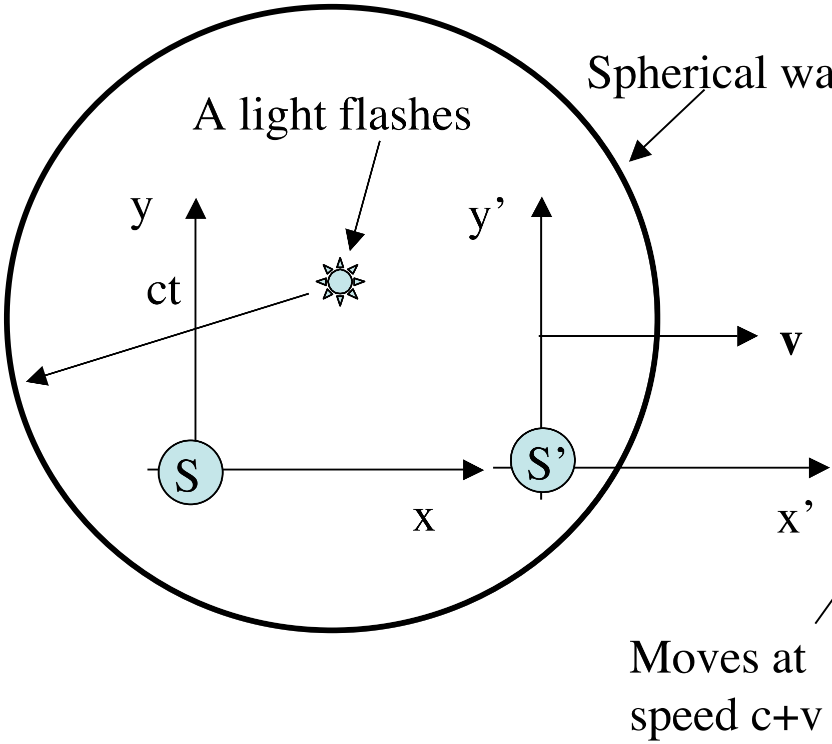
Which of these is in an inertial reference frame (or a very good approximation)?

A. A rocket being launched
B. A car rolling down a steep hill
C. A sky diver falling at terminal speed
D. A roller coaster going over the top of a hill
E. None of the above

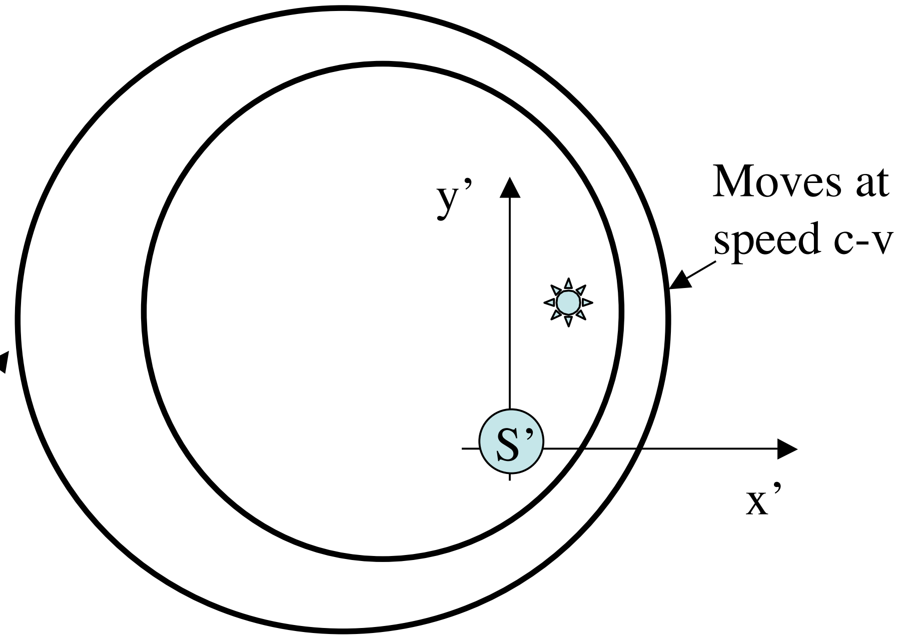
Ocean waves are approaching the beach at 10 m/s. A boat heading out to sea travels at 6 m/s. How fast are the waves moving in the boat's reference frame?

A. 4 m/s
B. 6 m/s
C. 16 m/s
D. 10 m/s

Maxwell's Equations seem to imply that there is a preferred reference frame



If Galilean transformations apply a spherical wave spreads at from a moving point.



Question: A light flashes. Observer S say's a spherical wave propagates away from the point of the flash. What does Observer S' say?

Same as propagation of waves in a medium - The ether. All attempts to measure the ether failed.

Using the interferometer Michelson and Morley showed that the speed of light is independent of the motion of the earth.

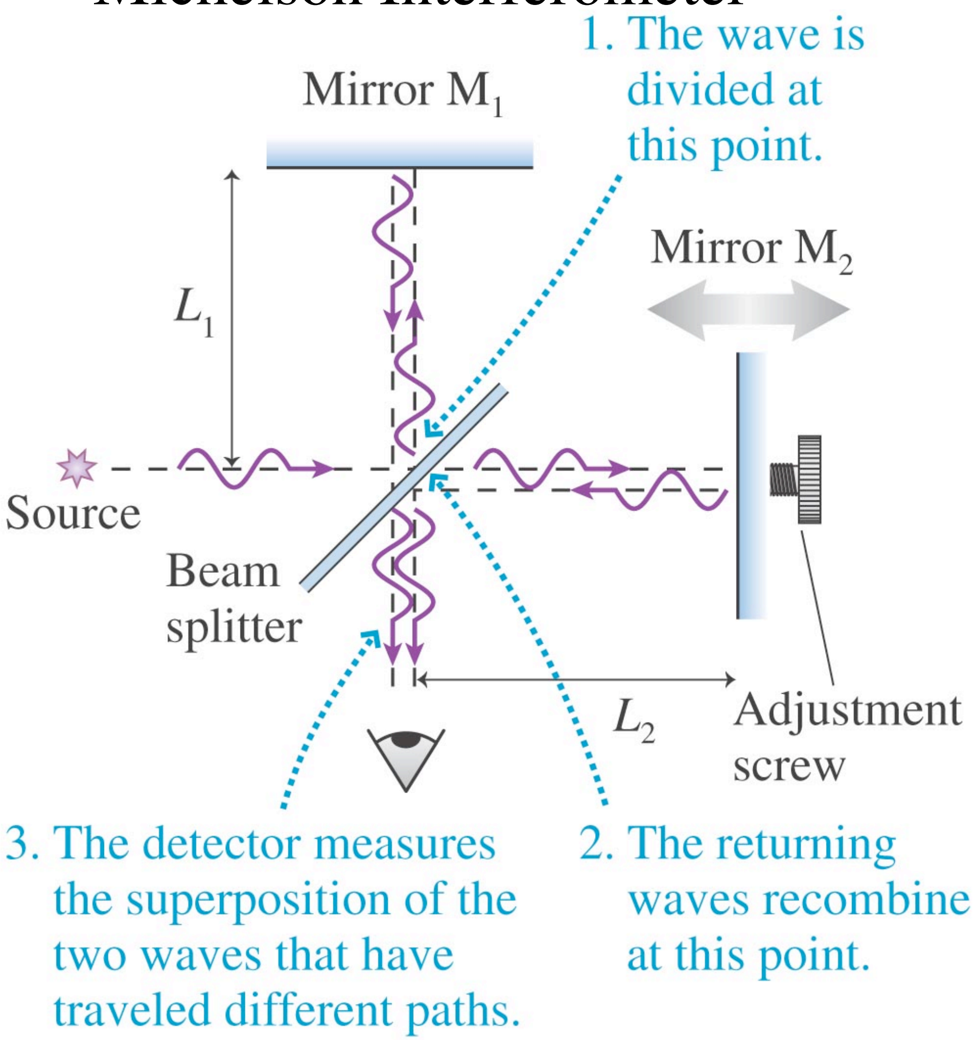
This implies that light is not supported by a medium, but propagates in vacuum.

Led to development of the special theory of relativity.

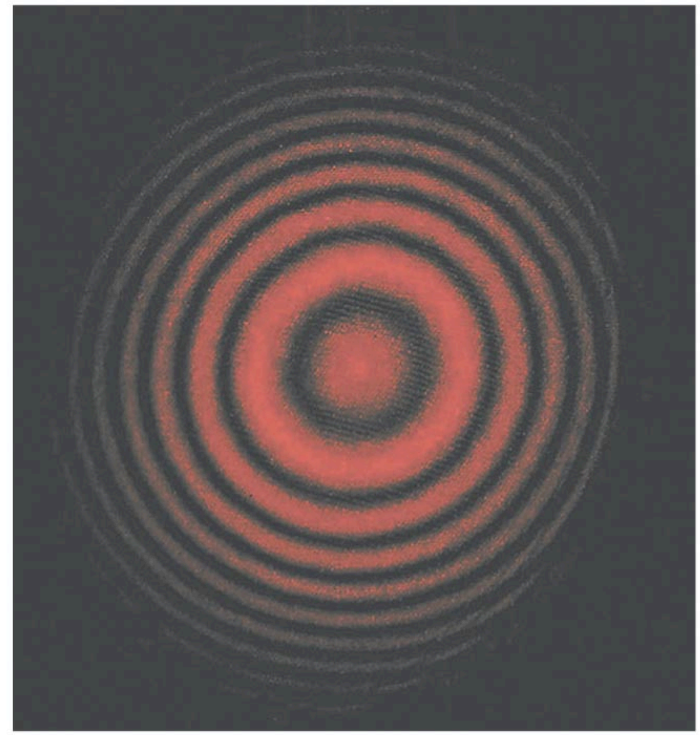
Albert Michelson
First US Nobel Science Prize Winner

Wikimedia Commons

Michelson Interferometer



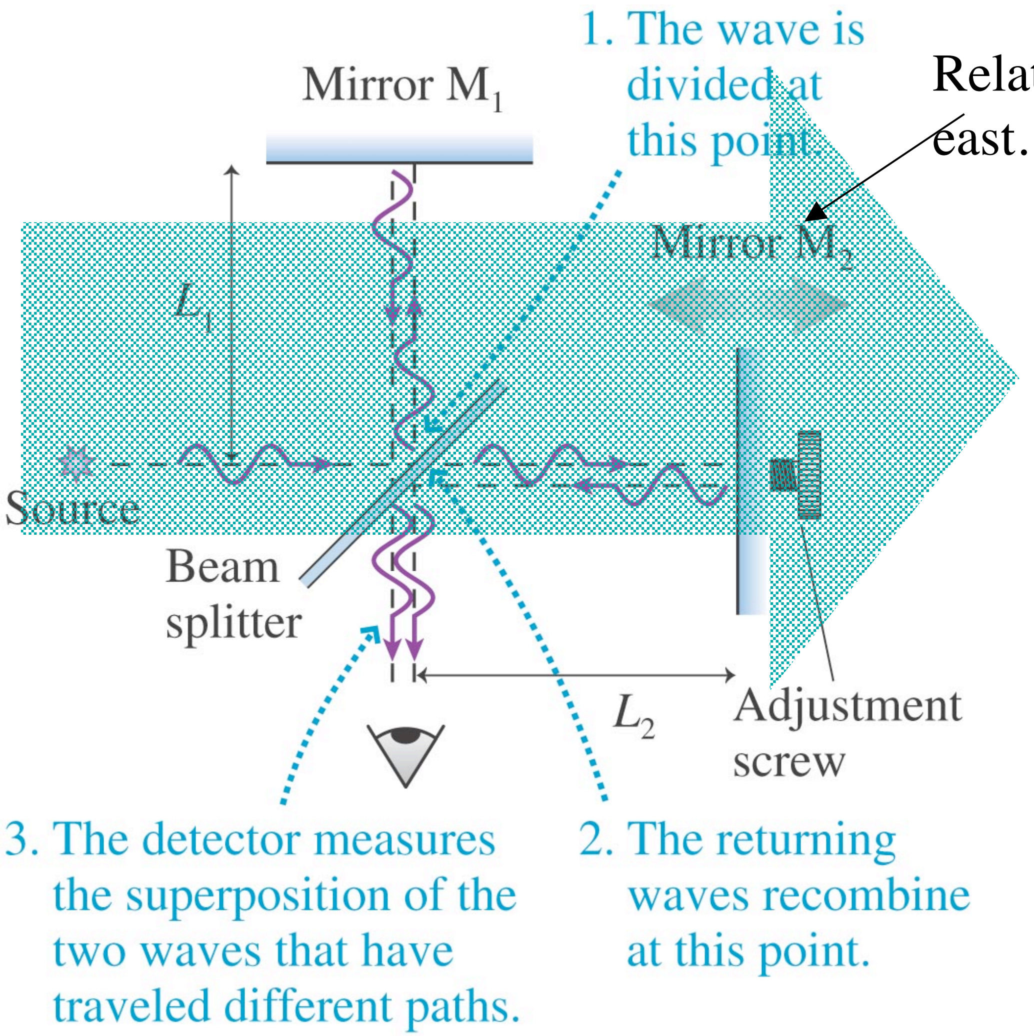
What is seen



As L2 is varied, central spot changes from dark to light, etc. Count changes = Δm

If I vary L_2 $$\Delta m = \frac{\Delta L_2}{\lambda / 2}$$

Measuring Index of refraction



Relative motion of ether, v_e, west to east.

Travel time on leg 2

$$t_2 = \frac{L_2}{c+v_e} + \frac{L_2}{c-v_e} = \frac{2L_2}{c(1-v_e^2/c^2)}$$

Travel time on leg 1

$$t_1 = \frac{L_1}{c\sqrt{1-v_e^2/c^2}}$$

Turn adjustment screw until constructive interference occurs. Then rotate whole experiment so that Leg 1 is now east to west. Interference should change if ether is present. It doesn't. Speed of light is the same north-south as east-west.

Einstein's Postulates

1. All the laws of physics are the same in all inertial reference frames

   That the laws are the same does not mean that the values of the measured quantities will be the same. The rules are the same.

2. The speed of light is the same for all observers

   There is no ether.

These postulates require that we replace Galilean transformations with something else - Lorentz transformations.

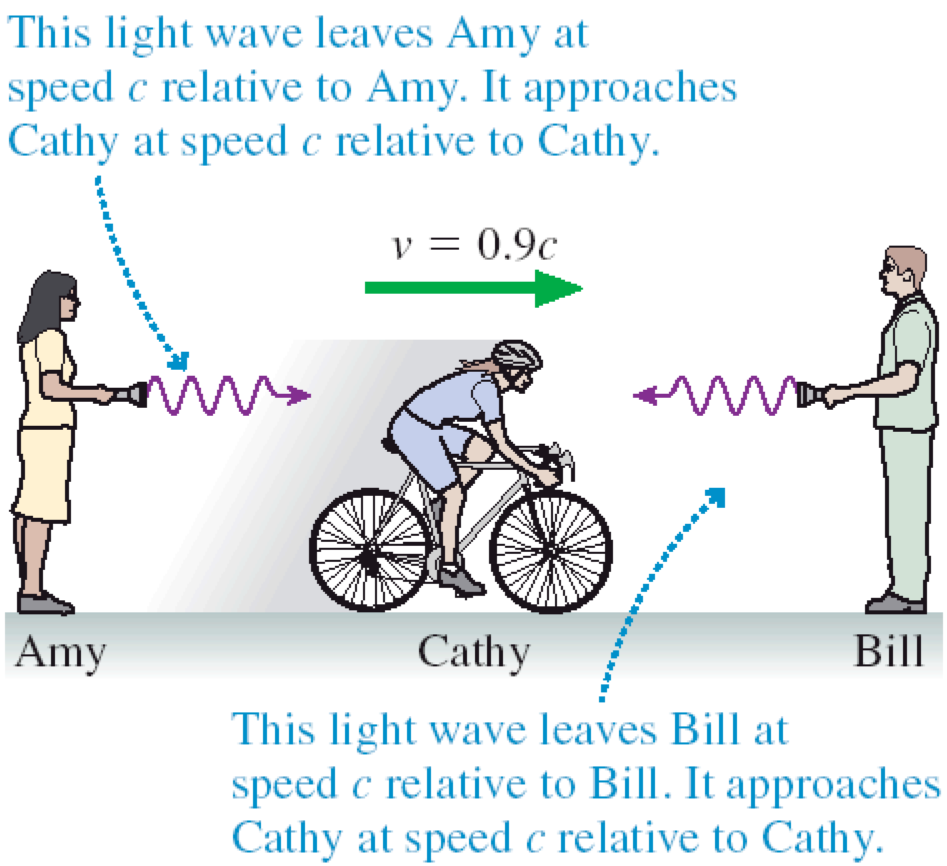
Einstein's Principle of Relativity

Principle of relativity All the laws of physics are the same in all inertial reference frames.

- Maxwell's equations are true in all inertial reference frames.
- Maxwell's equations predict that electromagnetic waves, including light, travel at speed $c = 3.00 \times 10^8$ m/s.
- Therefore, **light travels at speed *c* in all inertial reference frames.**

Every experiment has found that light travels at 3.00×10^8 m/s in every inertial reference frame, regardless of how the reference frames are moving with respect to each other.

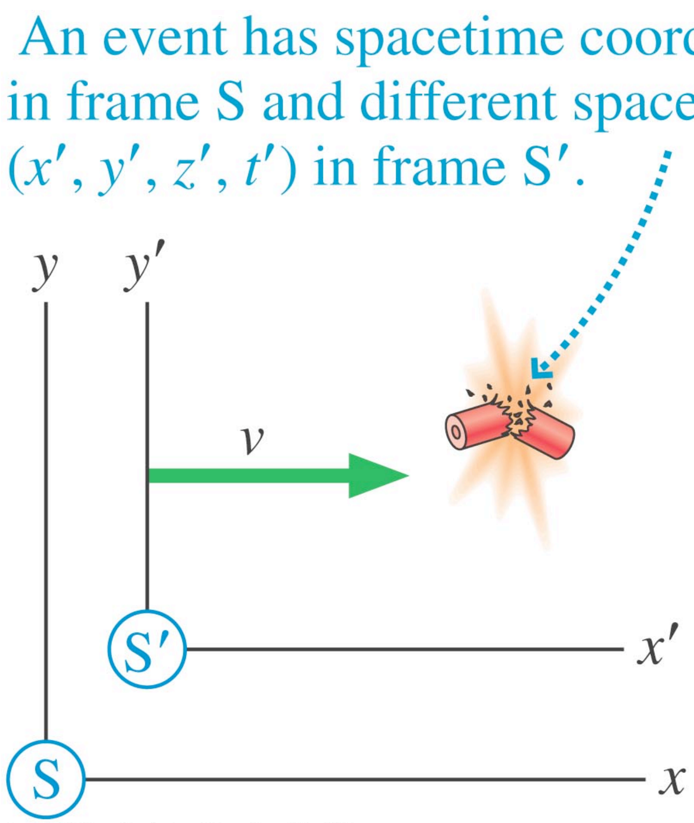
FIGURE 37.9 Light travels at speed c in all inertial reference frames, regardless of how the reference frames are moving with respect to the light source.



Events

In order to describe the way coordinates and time in one frame are related to coordinates in time in another we need to start thinking in terms of **events**.

An event is something that happens at a particular point in space and at a particular time.



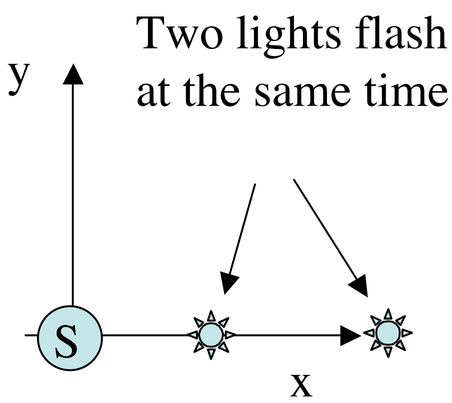
An event has four coordinates
3 space + time.

The time represents the actual time the event occurred, **not** the time the information about the event arrived at some detector. We assume we can design detectors that can determine the actual time..

A carpenter is working on a house two blocks away. You notice a slight delay between seeing the carpenter's hammer hit the nail and hearing the blow. At what time does the event "hammer hits nail" occur?

A. Very slightly after you see the hammer hit.
B. Very slightly after you hear the hammer hit.
C. Very slightly before you see the hammer hit.
D. At the instant you hear the blow.
E. At the instant you see the hammer hit.

Lack of simultaneity



Two lights flash at the same time - t=2s.
Light #1 is at the point (x=2m, y=0, z=0).
Light #2 is at the point (x=4m, y=0, z=0).

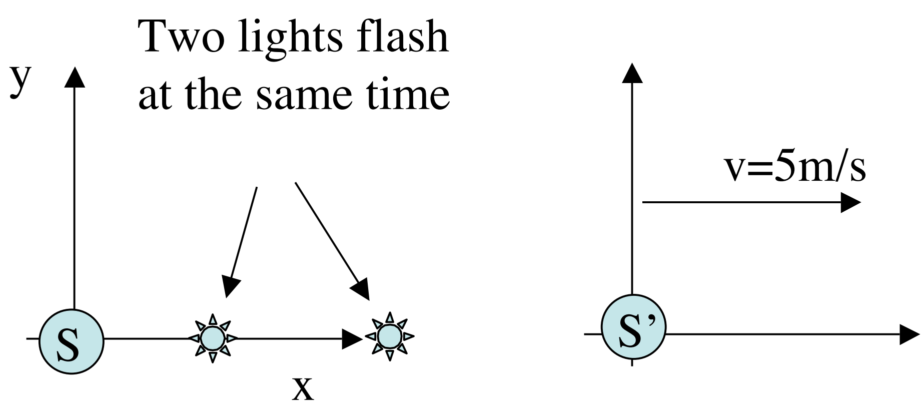
What are the space-time coordinates of event #1?

What are the space-time coordinates of event #2?

Suppose light is detected at the origin.

When does it arrive?

Does this change your answer for the space time coordinates?



Apply a <u>Galilean</u> transformation to find the space time coordinates of the two events in the frame S'

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

In S:

Event #1

$$x_1 = 2$$
$$y_1 = 0$$
$$z_1 = 0$$
$$t_1 = 2$$

Event #2

$$x_2 = 4$$
$$y_2 = 0$$
$$z_2 = 0$$
$$t_2 = 2$$

In S':

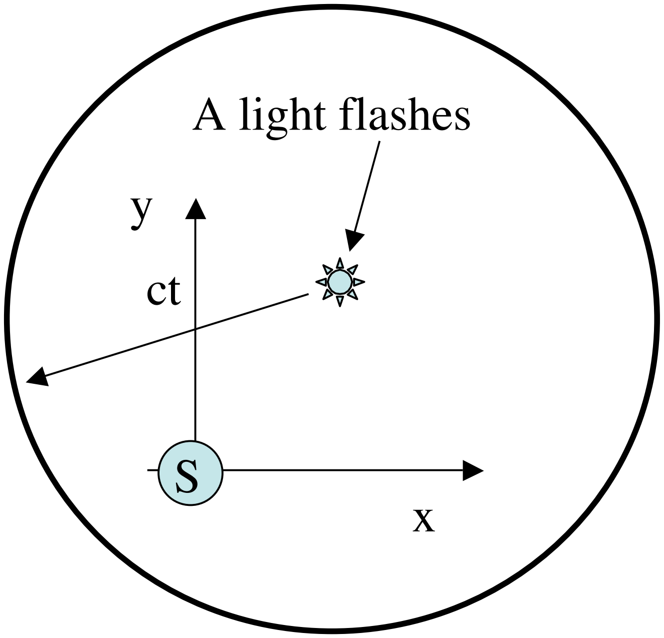
Event #1

$$x_1' = 2 - 8 \cdot 2 = -8$$
$$y_1' = 0$$
$$z_1' = 0$$
$$t_1' = 2$$

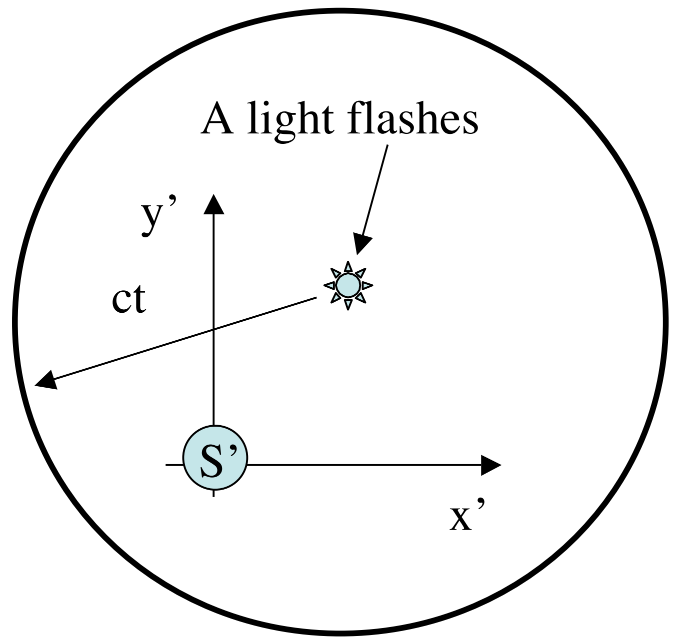
Event #2

$$x_2' = 4 - 2 \cdot 5 = -6$$
$$y_2' = 0$$
$$z_2' = 0$$
$$t_2' = 2$$

Note: spatial distance between events is the same in both frames and time events occur is the same in both frames. Neither of these will be true when we consider Lorenz transformations.



Lorentz transformations of space and time are such that all observers see a spherical wave front propagating at c.



The biggest conceptual difficulty is that two things that happen at the same time in one frame, happen at different times in another frame.

	Galilean	Lorentz
Transformation	$x' = x - vt$ $y' = y$ $z' = z$ $t' = t$	$x' = \gamma\left(x - vt\right)$ $y' = y$ $z' = z$ $t' = \gamma\left(t - vx / c^2\right)$ $\gamma = 1 / \sqrt{1 - (v / c)^2}$
Inverse	$x = x' + vt'$ $y = y'$ $z = z'$ $t = t'$	$x = \gamma\left(x' + vt'\right)$ $y = y$ $z = z$ $t = \gamma\left(t' + vx' / c^2\right)$ $\gamma = 1 / \sqrt{1 - (v / c)^2}$

Comments

$$x' = \gamma(x - vt)$$

Two events occurring at the same time in S, but separated in space will appear to be further separated in S'- (space contraction)

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - vx / c^2\right)$$

Two events occurring at the same time in S, but separated in space will not occur at the same time S'

$$\gamma = 1 / \sqrt{1 - (v / c)^2} > 1$$

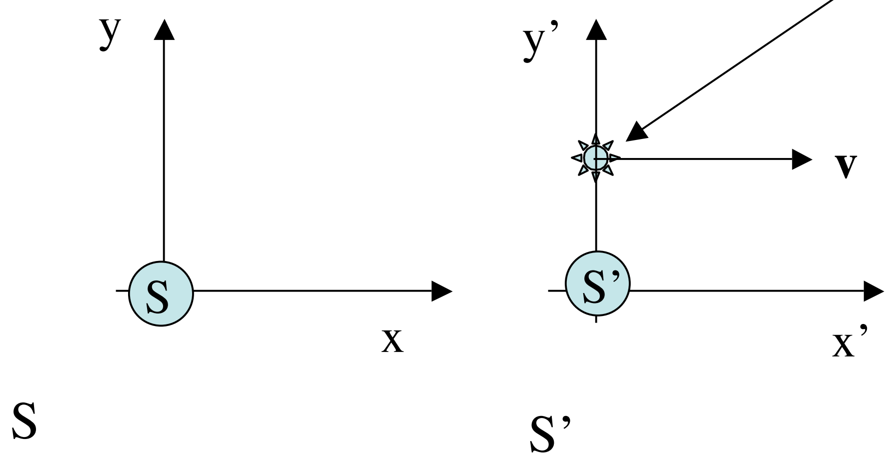
Time dilation and length contraction.

Time for moving objects appears to slow down for a stationary observer.

Length of a moving object appears to contact for a stationary observer.

Time Dilation

A moving light flashes at regular intervals T’ in its own frame S’ (rest frame). It’s a clock.



Event #1 - first flash $x_1' = 0$ $t_1' = 0$

Event #2 - second flash $x_2' = 0$ $t_2' = T'$

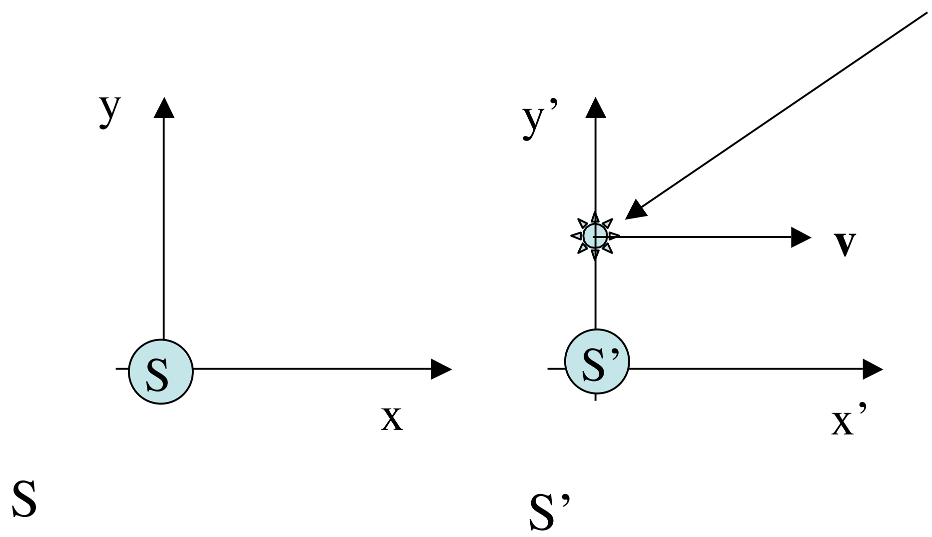
Calculate the coordinates of the two flashes in S.

Which Transformation should I use?

	Galilean	Lorentz
Transformation	$x' = x - vt$ $y' = y$ $z' = z$ $t' = t$	$x' = \gamma\left(x - vt\right)$ $y' = y$ $z' = z$ $t' = \gamma\left(t - vx / c^2\right)$ $\gamma = 1 / \sqrt{1 - (v / c)^2}$
Inverse	$x = x' + vt'$ $y = y'$ $z = z'$ $t = t'$	$x = \gamma\left(x' + vt'\right)$ $y = y$ $z = z$ $t = \gamma\left(t' + vx' / c^2\right)$ $\gamma = 1 / \sqrt{1 - (v / c)^2}$

Time Dilation

A moving light flashes at regular intervals T' in its own frame S' (rest frame). It's a clock.



Event #1 - first flash $\quad x_1' = 0$, $t_1' = 0$

Event #2 - second flash $\quad x_2' = 0$, $t_2' = T'$

Calculate the coordinates of the two flashes in S.

Event #1
- first flash

$$x_1 = \gamma\left(x_1' + vt_1'\right) = 0$$
$$t_1 = \gamma\left(t_1' + vx_1' / c^2\right) = 0$$

Event #2
- second flash

$$x_2 = \gamma\left(x_2' + vt_2'\right) = \gamma v T'$$
$$t_2 = \gamma\left(t_2' + vx_2' / c^2\right) = \gamma T'$$

In S period between flashes is

$$\gamma T' > T'$$

Clock appears to run slow

$$\gamma = 1 / \sqrt{1 - (v / c)^2}$$

Proper Time

The time between two events that occur at the same point in space is called the **proper time**. Label proper time $\Delta\tau$

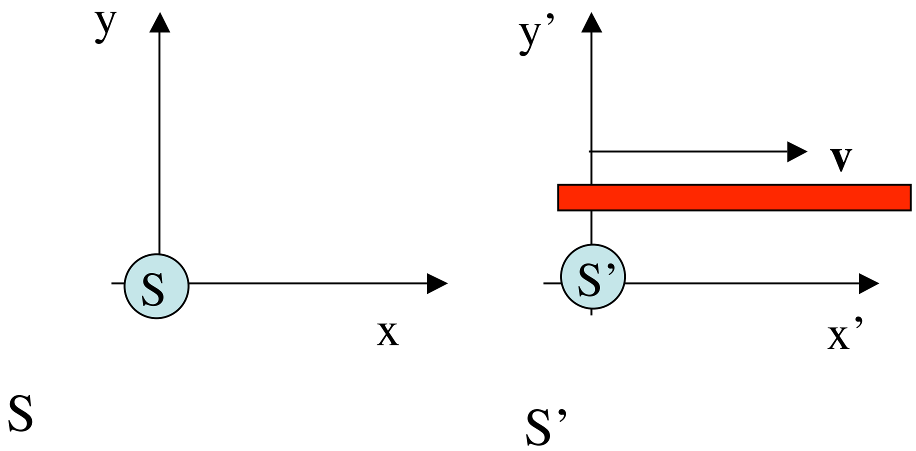
In some other reference frame these events will occur at different points in space. They will be separated in time by a time interval Δt.

$$\Delta t = \gamma \Delta \tau$$

$$\gamma = 1/\sqrt{1-(v/c)^2}$$
$$\gamma = 1/\sqrt{1-\beta^2}$$
$$\beta = v/c$$

$$\Delta \tau = \Delta t / \gamma = \sqrt{1-\beta^2}\,\Delta t$$

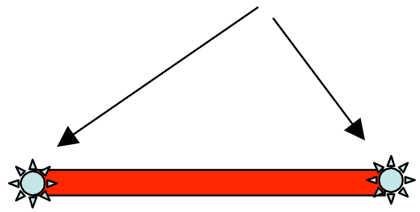
Space Contraction



A bar of length L' in its own frame (S') is moving with velocity v relative to an observer in frame S. What length does the bar have in S?

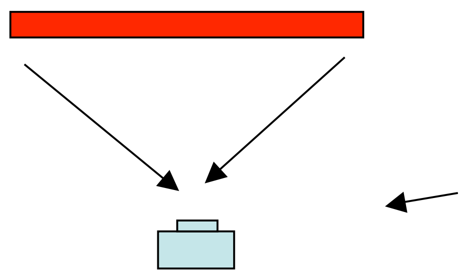
We need two events.
What two events should we pick?

Two flashing lights, one on each end of the bar.



When should they flash?

or



Camera takes snap shot as bar goes by.

Same path length from ends to camera

The two events should occur at the same time in S.
Gives the length of the object in S.

In S

Event #1 - left flash

$$x_1 = 0$$
$$t_1 = 0$$

Event #2 - right flash

$$x_2 = L$$
$$t_2 = 0$$

In S'

$$x_1' = \gamma(x_1 - vt_1) = 0$$
$$t_1' = \gamma\left(t_1 - vx_1 / c^2\right) = 0$$

$$x_2' = \gamma(x_2 - vt_2) = \gamma L$$
$$t_2' = \gamma\left(t_2 - vx_2 / c^2\right) = \gamma\left(-vL / c^2\right)$$

Length in S'

$$L' = \gamma L$$
$$L' > L$$

Bar is shorter in S

In S' the bar is stationary, so the fact that the two events occur at different times in S' is not important.

Length Contraction

The distance L' between two objects, or two points on one object, measured in the reference frame in which the objects are at rest is called the **proper length**. The distance L in a reference frame S is

$$L = L' / \gamma = \sqrt{1-\beta^2}\, L' \qquad \beta = v / c$$

$$L' > L$$

NOTE: Length contraction does not tell us how an object would *look*. The visual appearance of an object is determined by light waves that arrive simultaneously at the eye. Length and length contraction are concerned only with the *actual* length of the object at one instant of time.

A tree and a pole are 3000 m apart. Each is suddenly hit by a bolt of lightning. Mark, who is standing at rest midway between the two, sees the two lightning bolts at the same instant of time. Nancy is at rest under the tree. Define event 1 to be "lightning strikes tree" and event 2 to be "lightning strikes pole." For Nancy, does event 1 occur before, after or at the same time as event 2?

A. before event 2
B. after event 2
C. at the same time as event 2

A tree and a pole are 3000 m apart. Each is suddenly hit by a bolt of lightning. Mark, who is standing at rest midway between the two, sees the two lightning bolts at the same instant of time. Rachel is flying Nancy's rocket at $v = 0.5c$ in the direction from the tree toward the pole. The lightning hits the tree just as she passes by it. Define event 1 to be "lightning strikes tree" and event 2 to be "lightning strikes pole." For Rachel, does event 1 occur before, after or at the same time as event 2?

A. before event 2
B. after event 2
C. at the same time as event 2

	Event #1: Tree	Event #2: Pole
Mark S:	$x_1 = 0$ $t_1 = 0$	$x_2 = L = 3000m$ $t_2 = 0$
Rachel S':	$x_1' = \gamma(x_1 - vt_1) = 0$ $t_1' = \gamma\left(t_1 - vx_1 / c^2\right) = 0$	$x_2' = \gamma(x_2 - vt_2) = \gamma(L)$ $t_2' = \gamma\left(t_2 - vx_2 / c^2\right)$ $= \gamma\left(-vL / c^2\right) < t_1'$

Event 2 occurs before Event 1 in S'

$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - vx / c^2\right)$$

EXAMPLE 37.5 From the sun to Saturn

QUESTIONS:

EXAMPLE 37.5 From the sun to Saturn

Saturn is 1.43×10^{12} m from the sun. A rocket travels along a line from the sun to Saturn at a constant speed of $0.9c$ relative to the solar system. How long does the journey take as measured by an experimenter on earth? As measured by an astronaut on the rocket?

EXAMPLE 37.5 From the sun to Saturn

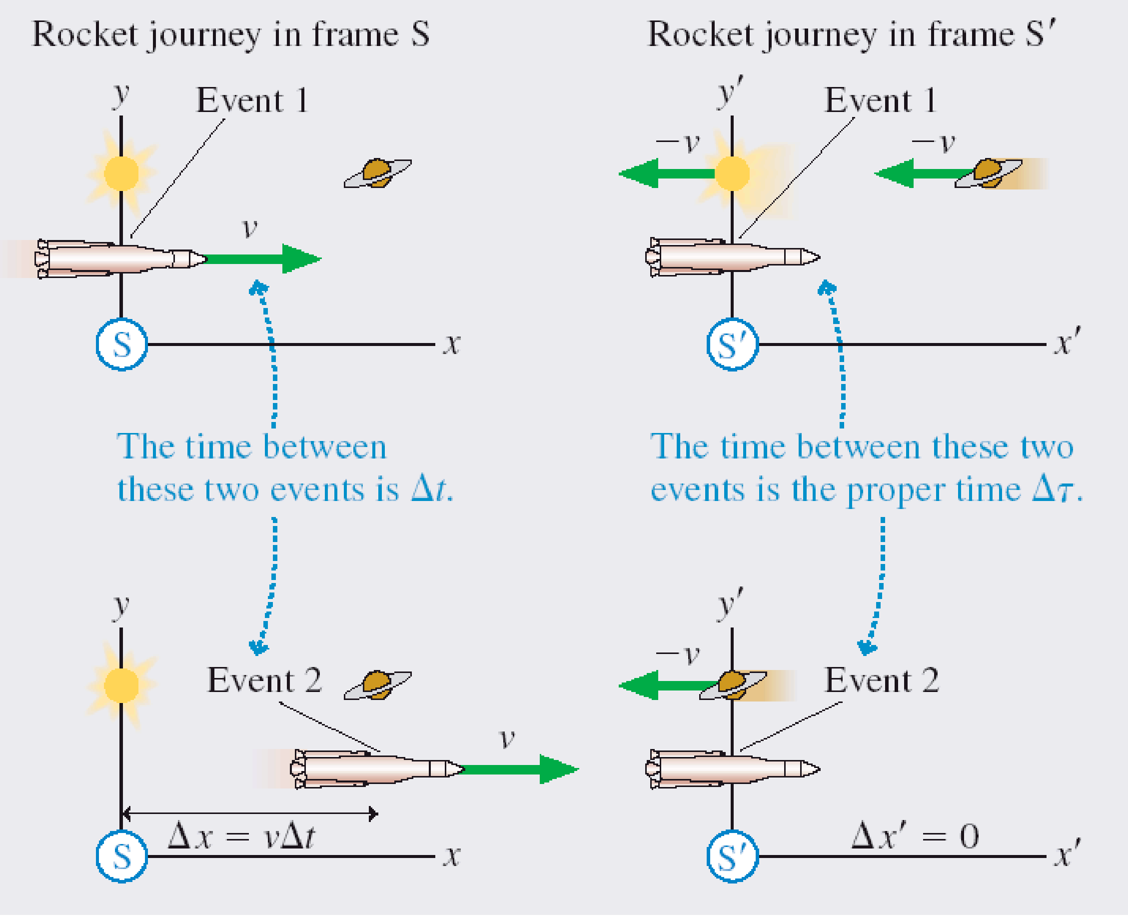
MODEL Let the solar system be in reference frame S and the rocket be in reference frame S′ that travels with velocity $v = 0.9c$ relative to S. Relativity problems must be stated in terms of *events*. Let event 1 be “the rocket and the sun coincide” (the experimenter on earth says that the rocket passes the sun; the astronaut on the rocket says that the sun passes the rocket) and event 2 be “the rocket and Saturn coincide.”

EXAMPLE 37.5 From the sun to Saturn

VISUALIZE **FIGURE 37.22** shows the two events as seen from the two reference frames. Notice that the two events occur at the *same position* in S′, the position of the rocket, and consequently can be measured by *one* clock.

EXAMPLE 37.5 From the sun to Saturn

FIGURE 37.22 Pictorial representation of the trip as seen in frames S and S′.



EXAMPLE 37.5 From the sun to Saturn

SOLVE The time interval measured in the solar system reference frame, which includes the earth, is simply

$$\Delta t = \frac{\Delta x}{v} = \frac{1.43 \times 10^{12}\ \text{m}}{0.9 \times (3.00 \times 10^{8}\ \text{m/s})} = 5300\ \text{s}$$

Relativity hasn't abandoned the basic definition $v = \Delta x/\Delta t$, although we do have to be sure that Δx and Δt are measured in just one reference frame and refer to the same two events.

EXAMPLE 37.5 From the sun to Saturn

How are things in the rocket's reference frame? The two events occur at the *same position* in S′ and can be measured by *one* clock, the clock at the origin. Thus the time measured by the astronauts is the *proper time* $\Delta\tau$ between the two events. We can use Equation 37.9 with $\beta = 0.9$ to find

$$\Delta\tau = \sqrt{1 - \beta^2}\,\Delta t = \sqrt{1 - 0.9^2}\,(5300\ \text{s}) = 2310\ \text{s}$$

EXAMPLE 37.5 From the sun to Saturn

ASSESS The time interval measured between these two events by the astronauts is less than half the time interval measured by experimenters on earth. The difference has nothing to do with when earthbound astronomers *see* the rocket pass the sun and Saturn. Δt is the time interval from when the rocket actually passes the sun, as measured by a clock at the sun, until it actually passes Saturn, as measured by a synchronized clock at Saturn. The interval between *seeing* the events from earth, which would have to allow for light travel times, would be something other than 5300 s. Δt and $\Delta\tau$ are different because *time is different* in two reference frames moving relative to each other.

Space-Time Invariant

Consider two events which are separated in space and time

Separation in S
$$\begin{cases} \Delta x \\ \Delta y \\ \Delta z \\ \Delta t \end{cases}$$

Separation in S'
$$\begin{cases} \Delta x' = \gamma\left(\Delta x - v\Delta t\right) \\ \Delta y' = \Delta y \\ \Delta z' = \Delta z \\ \Delta t' = \gamma\left(\Delta t - v\Delta x / c^2\right) \end{cases}$$

You can show

$$c^2\Delta t^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2)$$
$$= c^2\Delta t'^2 - (\Delta x'^2 + \Delta y'^2 + \Delta z'^2)$$

Space time interval is the same for all observers

Consequence: We know it is possible for two events to occur in different order depending the reference frame in which they are viewed

Space time interval is the same for all observers

$$s^2 \equiv c^2\Delta t^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2)$$

Consequences: We know it is possible for two events to occur in a different order and at different places depending the reference frame in which they are viewed.

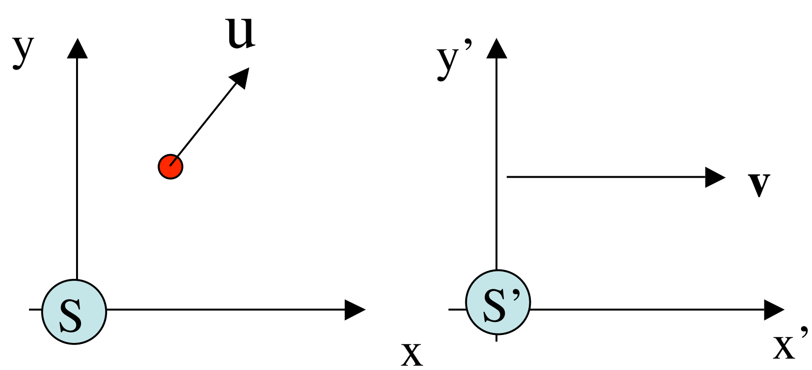
But:
If $s^2>0$ then

There is a reference frame where the two events occur at the same place.
The two events will always occur at different times.
The order of the events will be the same in all frames.
It is possible that the first event caused the second.

If $s^2<0$ then

There is a reference frame where the two events occur at the same time.
The two events can not occur at the same place.
Neither event could have caused the other.

Relativistic transformation of velocity



Ball has velocity **u** in S, what is velocity **u**' in S'

Coordinates of ball

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - vx / c^2\right)$$

With time each changes

$$dx' = \gamma(dx - vdt)$$
$$dy' = dy$$
$$dz' = dz$$
$$dt' = \gamma\left(dt - vdx / c^2\right)$$

$$u_x' \equiv \frac{dx'}{dt'} = \frac{\gamma(dx - vdt)}{\gamma\left(dt - vdx / c^2\right)}$$

$$u_x' = \frac{(dx / dt - v)}{\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)} = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

$$\boxed{u_x' = \frac{u_x - v}{1 - \frac{vu_x}{c^2}} \qquad u_y' = \frac{u_y}{\gamma(1 - \frac{vu_x}{c^2})}}$$

Relativistic transformation of velocity

$$u'_x = \frac{u_x - v}{1 - vu_x / c^2}$$

$$u'_y = \frac{u_y}{\gamma(1 - vu_x / c^2)}$$

$$u'_z = \frac{u_z}{\gamma(1 - vu_x / c^2)}$$

Special cases

1. Nonrelativistic motion,
   recover Galilean Transformation

$$u_x, v << c$$

$$u_x{}' = u_x - v$$

$$u'_y = u_y$$

$$u'_z = u_z$$

2. Speed of light

$$u_x = c$$

$$u_y = 0$$

$$u_z = 0$$

$$u_x{}' = \frac{c - v}{1 - v / c} = c$$

Can you show that if $u_x^2 + u_y^2 + u_z^2 = c^2$

Then

$$u_x'^2 + u_y'^2 + u_z'^2 = c^2$$

EXAMPLE 37.10 A really fast bullet

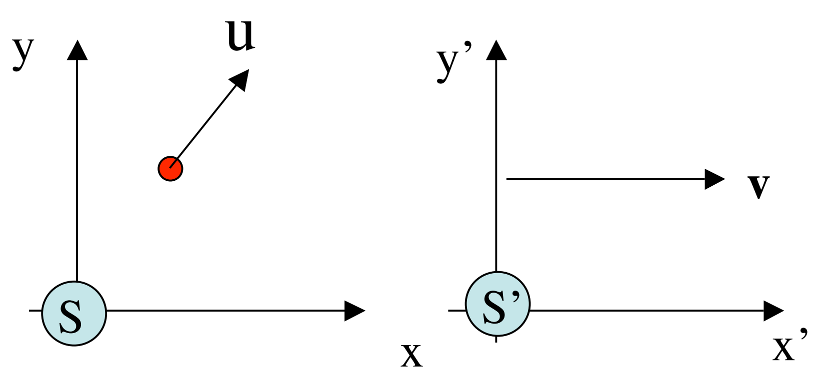
QUESTION:

EXAMPLE 37.10 A really fast bullet

A rocket flies past the earth at $0.90c$. As it goes by, the rocket fires a bullet in the forward direction at $0.95c$ with respect to the rocket. What is the bullet's speed with respect to the earth?

EXAMPLE 37.10 A really fast bullet

MODEL The rocket and the earth are inertial reference frames. Let the earth be frame S and the rocket be frame S′. The velocity of frame S′ relative to frame S is $v = 0.90c$. The bullet's velocity in frame S′ is $u' = 0.95c$.



transformation

$$u'_x = \frac{u_x - v}{1 - \dfrac{vu_x}{c^2}}$$

inverse

$$u_x = \frac{u'_x + v}{1 + \dfrac{vu'_x}{c^2}}$$

EXAMPLE 37.10 A really fast bullet

SOLVE We can use the Lorentz velocity transformation to find

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.95c + 0.90c}{1 + (0.95c)(0.90c)/c^2} = 0.997c$$

NOTE ► Many relativistic calculations are much easier when velocities are specified as a fraction of c. ◄

EXAMPLE 37.10 A really fast bullet

ASSESS In Newtonian mechanics, the Galilean transformation of velocity would give $u = 1.85c$. Now, despite the very high speed of the rocket and of the bullet with respect to the rocket, the bullet's speed with respect to the earth remains less than c. This is yet more evidence that objects cannot exceed the speed of light.

Relativistic Momentum

The momentum of a particle moving at speed u is

$$\boxed{p = \gamma_p m u}$$

$$\gamma_p = \frac{1}{\sqrt{1 - u^2/c^2}}$$

where the subscript p indicates that this is γ for a particle, not for a reference frame.

- If $u \ll c$, the momentum approaches the Newtonian value of $p = mu$. As u approaches c, however, p approaches infinity.
- For this reason, a force cannot accelerate a particle to a speed higher than c, because the particle's momentum becomes infinitely large as the speed approaches c.

Where did this definition come from?

Old definition $$p_x = mu_x = m\frac{dx}{dt}$$

Replace dt by dt', time interval in frame in which particle is instantaneously at rest.

$$dt' = \gamma_p\left(dt - u_x dx / c^2\right) = dt / \gamma_p$$

$$\gamma_p = 1 / \sqrt{1 - u^2{}_x / c^2}$$

$$p_x = m\frac{dx}{dt'} = m\gamma_p \frac{dx}{dt}$$

Suppose I know momentum, what is velocity?

$$\vec{\mathbf{p}} = m\gamma_p \vec{\mathbf{u}} \quad \gamma_p = 1/\sqrt{1-u^2/c^2}$$

Square $\quad p^2 = m^2 \gamma^2{}_p u^2$

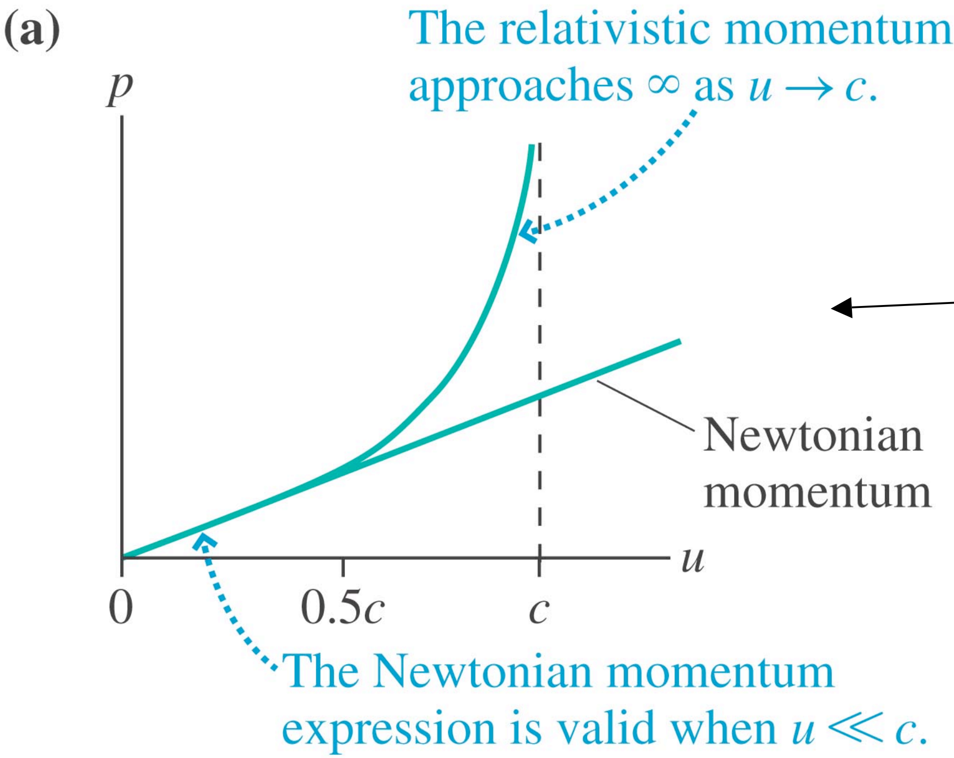
Then solve for

$$\frac{u^2}{c^2} = \frac{\left(p/mc\right)^2}{1+\left(p/mc\right)^2}$$

Solve for

$$\gamma_p = 1/\sqrt{1-u^2/c^2} = \sqrt{1+(p/mc)^2}$$

$$\vec{\mathbf{u}} = \frac{\vec{\mathbf{p}}}{m\gamma_p} \qquad \left|\mathbf{u}'\right| < c \qquad \text{Always}$$

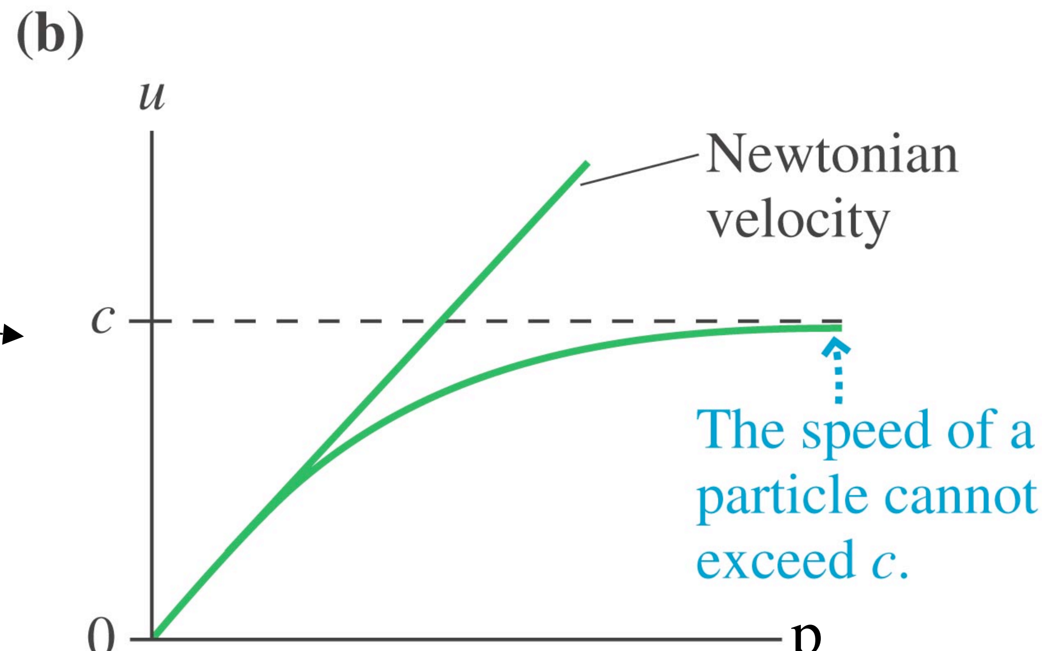


$$\vec{\mathbf{p}} = m\gamma_p \vec{\mathbf{u}}$$

$$\gamma_p = 1/\sqrt{1 - u^2/c^2}$$

$$\vec{\mathbf{u}} = \frac{\vec{\mathbf{p}}}{m\gamma_p}$$

$$\gamma_p = \sqrt{1 + (p/mc)^2}$$



EXAMPLE 37.11 Momentum of a subatomic particle

QUESTION:

EXAMPLE 37.11 Momentum of a subatomic particle

Electrons in a particle accelerator reach a speed of $0.999c$ relative to the laboratory. One collision of an electron with a target produces a muon that moves forward with a speed of $0.95c$ relative to the laboratory. The muon mass is 1.90×10^{-28} kg. What is the muon's momentum in the laboratory frame and in the frame of the electron beam?

EXAMPLE 37.11 Momentum of a subatomic particle

MODEL Let the laboratory be reference frame S. The reference frame S′ of the electron beam (i.e., a reference frame in which the electrons are at rest) moves in the direction of the electrons at $v = 0.999c$. The muon velocity in frame S is $u = 0.95c$.

EXAMPLE 37.11 Momentum of a subatomic particle

SOLVE γ_p for the muon in the laboratory reference frame is

$$\gamma_p = \frac{1}{\sqrt{1 - u^2/c^2}} = \frac{1}{\sqrt{1 - 0.95^2}} = 3.20$$

Thus the muon's momentum in the laboratory is

$$\begin{aligned} p = \gamma_p mu &= (3.20)(1.90 \times 10^{-28}\,\text{kg})(0.95 \times 3.00 \times 10^8\,\text{m/s}) \\ &= 1.73 \times 10^{-19}\,\text{kg m/s} \end{aligned}$$

The momentum is a factor of 3.2 larger than the Newtonian momentum mu. To find the momentum in the electron-beam reference frame, we must first use the velocity transformation equation to find the muon's velocity in frame S′:

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{0.95c - 0.999c}{1 - (0.95c)(0.999c)/c^2} = -0.962c$$

EXAMPLE 37.11 Momentum of a subatomic particle

In the laboratory frame, the faster electrons are overtaking the slower muon. Hence the muon's velocity in the electron-beam frame is negative. γ'_p for the muon in frame S$'$ is

$$\gamma'_p = \frac{1}{\sqrt{1 - u'^2/c^2}} = \frac{1}{\sqrt{1 - 0.962^2}} = 3.66$$

The muon's momentum in the electron-beam reference frame is

$$\begin{aligned} p' &= \gamma'_p m u' \\ &= (3.66)(1.90 \times 10^{-28}\,\text{kg})(-0.962 \times 3.00 \times 10^{8}\,\text{m/s}) \\ &= -2.01 \times 10^{-19}\,\text{kg m/s} \end{aligned}$$

EXAMPLE 37.11 Momentum of a subatomic particle

ASSESS From the laboratory perspective, the muon moves only slightly slower than the electron beam. But it turns out that the muon moves faster with respect to the electrons, although in the opposite direction, than it does with respect to the laboratory.

Relativistic Energy

The **total energy** E of a particle is

$$E = \gamma_p mc^2 = E_0 + K = \text{rest energy} + \text{kinetic energy}$$

This total energy consists of a **rest energy**

$$E_0 = mc^2$$

and a relativistic expression for the *kinetic energy*

$$K = (\gamma_p - 1)mc^2 = (\gamma_p - 1)E_0$$

This expression for the kinetic energy is very nearly $mu^2/2$ when $u << c$.

Where does this definition of energy come from?

$$\frac{d}{dt}\gamma_p mc^2 = mc^2 \frac{d}{dt}\sqrt{1+(p/mc)^2} = \frac{p}{m\sqrt{1+(p/mc)^2}}\frac{dp}{dt}$$

Thus,

$$\frac{d}{dt}\gamma_p mc^2 = \frac{p}{m\gamma_p}\frac{dp}{dt} = u\frac{dp}{dt} = uF$$

Rate at which work is done

Replaces kinetic energy

Energy of photons and particles now given by the same formula

For photons: $E = hf$ $\quad p = h/\lambda$ $\quad \longrightarrow \quad E = pc$

For particles: $E = \gamma_p mc^2 = mc^2\sqrt{1+(p/mc)^2} = c\sqrt{(mc)^2+(p)^2}$

Let $\quad m \to 0 \quad\quad E \to pc$

EXAMPLE 37.12 Kinetic energy and total energy

EXAMPLE 37.12 Kinetic energy and total energy

Calculate the rest energy and the kinetic energy of (a) a 100 g ball moving with a speed of 100 m/s and (b) an electron with a speed of $0.999c$.

EXAMPLE 37.12 Kinetic energy and total energy

MODEL The ball, with $u \ll c$, is a classical particle. We don't need to use the relativistic expression for its kinetic energy. The electron is highly relativistic.

EXAMPLE 37.12 Kinetic energy and total energy

SOLVE a. For the ball, with $m = 0.10$ kg,

$$E_0 = mc^2 = 9.0 \times 10^{15}\ \text{J}$$

$$K = \frac{1}{2}mu^2 = 500\ \text{J}$$

b. For the electron, we start by calculating

$$\gamma_p = \frac{1}{(1 - u^2/c^2)^{1/2}} = 22.4$$

Then, using $m_e = 9.11 \times 10^{-31}$ kg, we find

$$E_0 = mc^2 = 8.2 \times 10^{-14}\ \text{J}$$

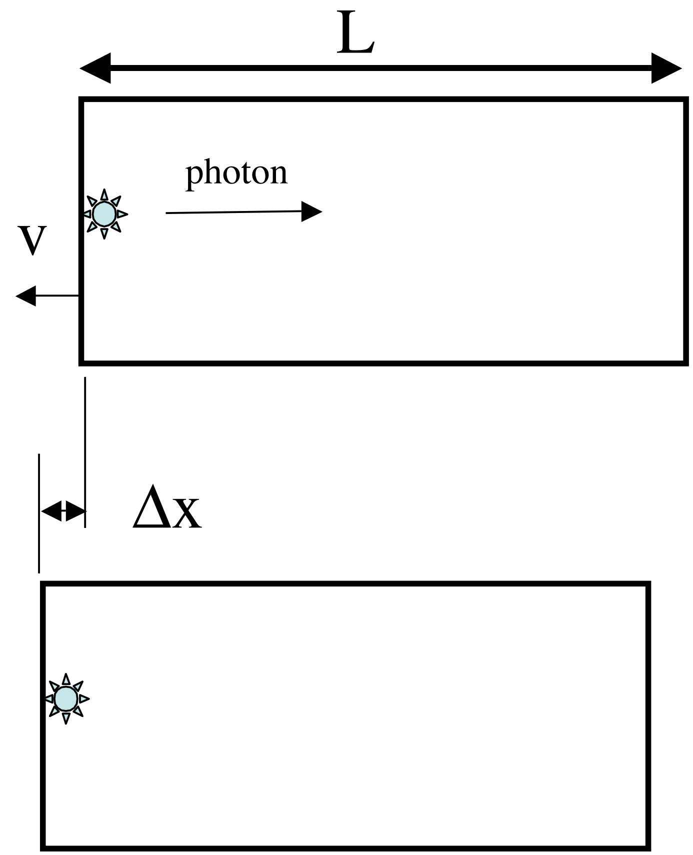
$$K = (\gamma_p - 1)E_0 = 170 \times 10^{-14}\ \text{J}$$

EXAMPLE 37.12 Kinetic energy and total energy

ASSESS The ball's kinetic energy is a typical kinetic energy. Its rest energy, by contrast, is a staggeringly large number. For a relativistic electron, on the other hand, the kinetic energy is more important than the rest energy.

Mass Energy Equivalence

Isolated box of mass M and length L in space.
A light on the wall on one side sends out a photon of energy E toward the right.
The photon has momentum $p=E/c$.
The box recoils with velocity $v=p/M$ to the left.
The photon is absorbed on the other side after a time $T=L/c$.
The box absorbs the momentum and stops moving.



Displacement of the box

$$\Delta x = vT = \frac{EL}{Mc^2}$$

Has the center of mass moved?
We would like to say no.
The box shouldn't be able to move its center of mass.

We can say that the CM hasn't moved if the photon reduced the mass of the left side by $m=E/c^2$ and increased the right side by the same amount.

$$E = mc^2$$

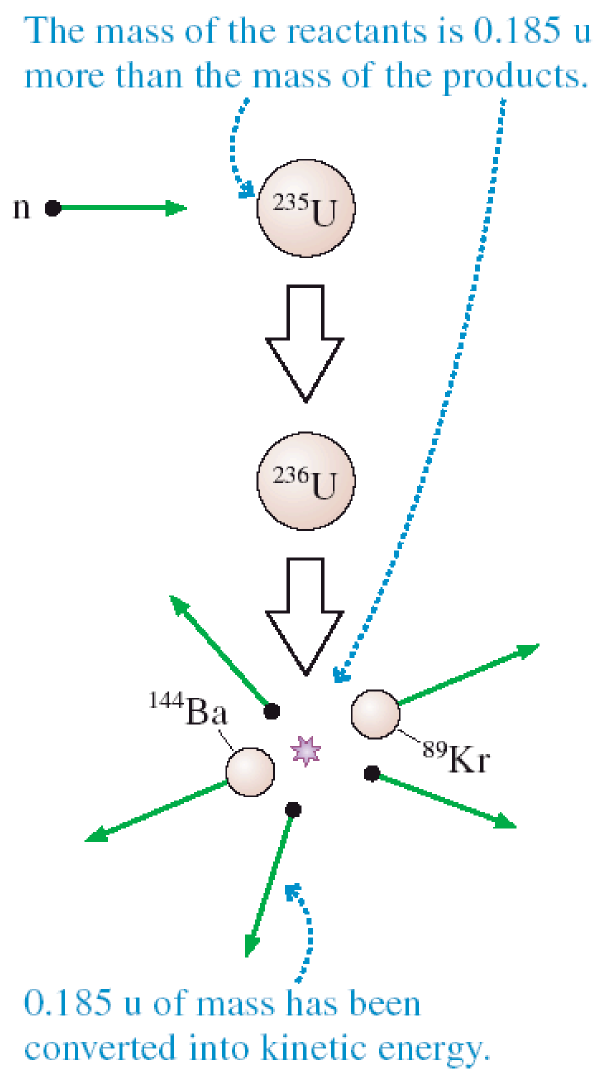
Conservation of Energy

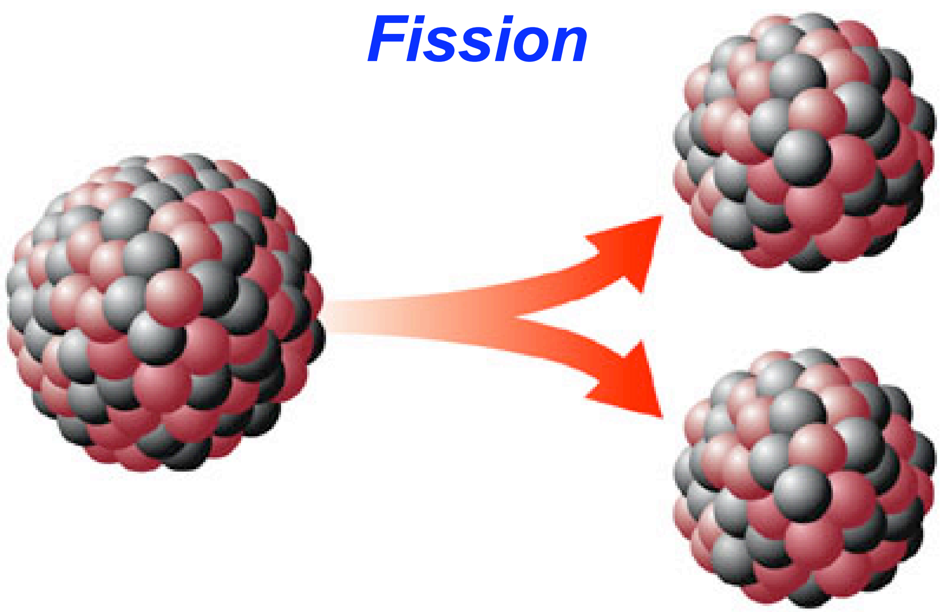
The creation and annihilation of particles with mass, processes strictly forbidden in Newtonian mechanics, are vivid proof that neither mass nor the Newtonian definition of energy is conserved. Even so, the *total* energy—the kinetic energy *and* the energy equivalent of mass—remains a conserved quantity.

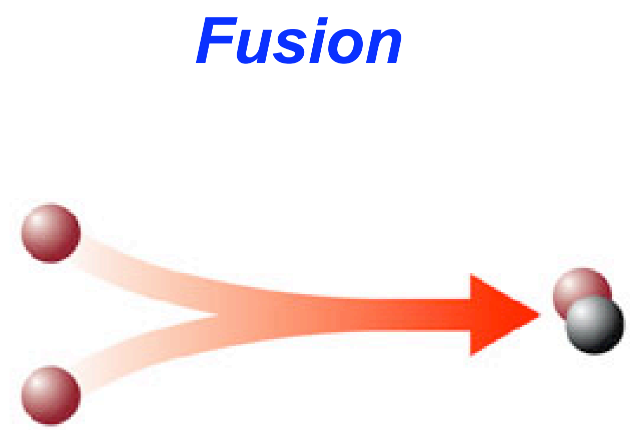
Law of conservation of total energy The energy $E = \sum E_i$ of an isolated system is conserved, where $E_i = (\gamma_p)_i m_i c^2$ is the total energy of particle i.

Mass and energy are not the same thing, but they are *equivalent* in the sense that mass can be transformed into energy and energy can be transformed into mass as long as the total energy is conserved.

FIGURE 37.41 In nuclear fission, the energy equivalent of lost mass is converted into kinetic energy.







Small nuclei stick together to make a bigger one and release energy

- Fusion powers all the stars, including the Sun.
- The fuel is hydrogen, but it has to be heated to millions of degrees to ignite the burn
- Power plants based on fusion could supply all our electrical needs. They could also be used to generate hydrogen for fuel cell cars, thus reducing consumption of oil.

General Principles

Principle of Relativity **All the laws of physics are the same in all inertial reference frames.**

- The speed of light c is the same in all inertial reference frames.
- No particle or causal influence can travel at a speed greater than c.

Important Concepts

Space

Spatial measurements depend on the motion of the experimenter relative to the events. An object's length is the difference between *simultaneous* measurements of the positions of both ends.

Proper length ℓ is the length of an object measured in a reference frame in which the object is at rest. The object's length in a frame in which the object moves with velocity v is

$$L = \sqrt{1 - \beta^2}\,\ell \leq \ell$$

This is called **length contraction.**

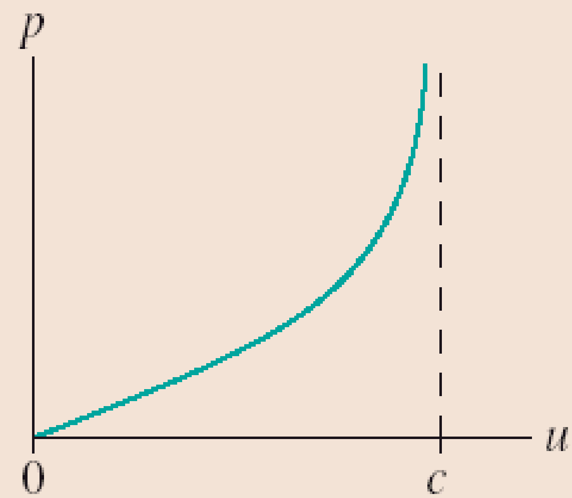
Important Concepts

Momentum

The law of conservation of momentum is valid in all inertial reference frames if the momentum of a particle with velocity u is $p = \gamma_p mu$, where

$$\gamma_p = 1/\sqrt{1 - u^2/c^2}$$

The momentum approaches ∞ as $u \rightarrow c$.



Important Concepts

Invariants are quantities that have the same value in all inertial reference frames.

Spacetime interval: $s^2 = (c\Delta t)^2 - (\Delta x)^2$

Particle rest energy: $E_0^2 = (mc^2)^2 = E^2 - (pc)^2$

Important Concepts

Time

Time measurements depend on the motion of the experimenter relative to the events. Events that are simultaneous in reference frame S are not simultaneous in frame S′ moving relative to S.

Proper time $\Delta\tau$ is the time interval between two events measured in a reference frame in which the events occur at the same position. The time interval between the events in a frame moving with relative velocity v is

$$\Delta t = \Delta\tau/\sqrt{1-\beta^2} \geq \Delta\tau$$

This is called **time dilation.**

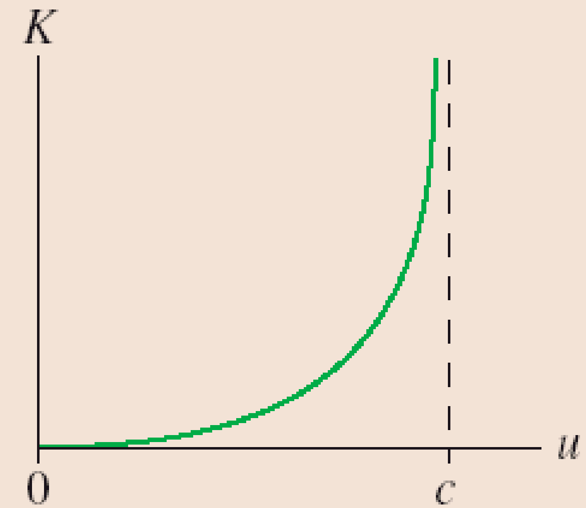
Important Concepts

Energy

The law of conservation of energy is valid in all inertial reference frames if the energy of a particle with velocity u is
$E = \gamma_p mc^2 = E_0 + K$

Rest energy $E_0 = mc^2$

Kinetic energy $K = (\gamma_p - 1)mc^2$.



Important Concepts

Mass-energy equivalence

Mass m can be transformed into energy $E = mc^2$.

Energy can be transformed into mass $m = \Delta E/c^2$.

Applications

An event happens at a specific place in space and time. Spacetime coordinates are (x, t) in frame S and (x', t') in frame S$'$.

Applications

A reference frame is a coordinate system with meter sticks and clocks for measuring events. Experimenters at rest relative to each other share the same reference frame.

Applications

The Lorentz transformations transform spacetime coordinates and velocities between reference frames S and S′.

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma(t - vx/c^2) \qquad t = \gamma(t' + vx'/c^2)$$
$$u' = \frac{u - v}{1 - uv/c^2} \qquad u = \frac{u' + v}{1 + u'v/c^2}$$

where u and u' are the x- and x'-components of velocity.

$$\beta = \frac{v}{c} \quad \text{and} \quad \gamma = 1/\sqrt{1 - v^2/c^2} = 1/\sqrt{1 - \beta^2}$$