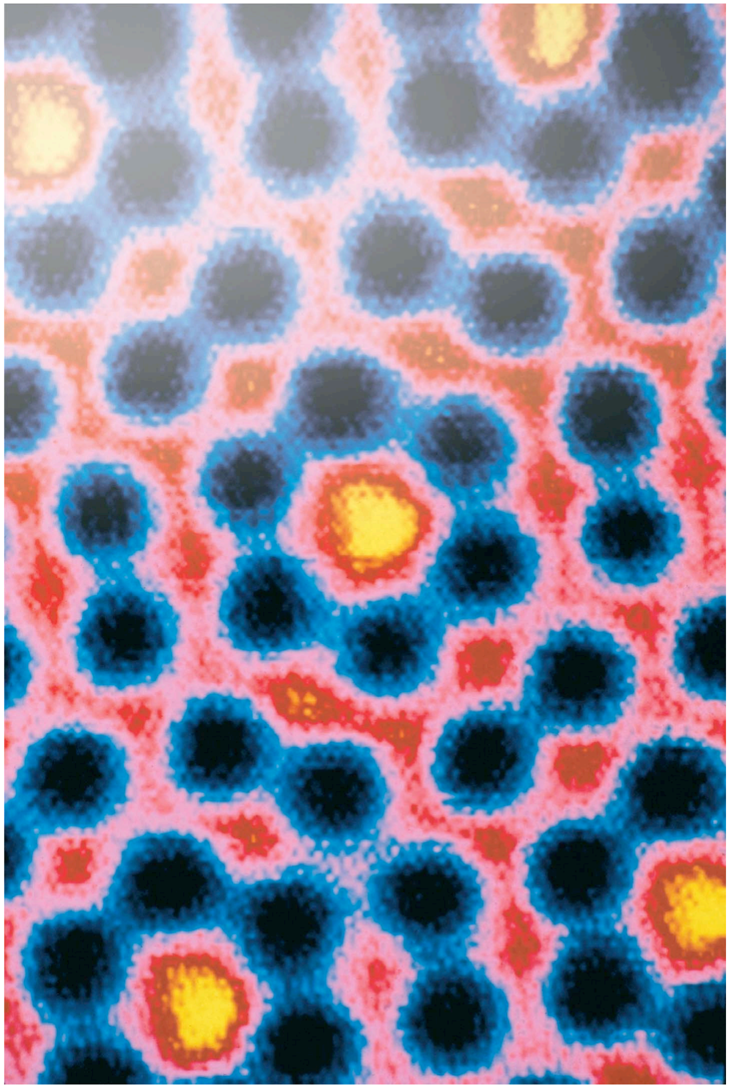
Chapter 25. Modern Optics and Matter Waves

This image of the individual atoms in a silicon crystal was made by exploiting the wave properties of electrons. Sometimes, electrons act less like particles and more like traveling waves. This is an important result of *quantum physics*.



Quantum Mechanics:

Basic Idea

Sometimes particles of matter behave as if they were some kind of wave.

Sometimes electromagnetic waves (light) behave as if they were composed of particles (photons)

In both cases an element of probability is introduced. We can no longer say what will happen in a set of circumstances, rather we can say what are the probabilities of various things happening.

Classical Physics

Newton's laws of motion + Maxwell's equations

Basically this is what you have learned so far and it does a good job explaining many things.

Things it can not explain:

- Discrete spectra of atoms
- Existence of atoms
- Photo-electric effect (photons)
- Diffraction of electrons

Classical physics was thought to describe the interaction of light with charges until there were discovered too many things that it could not explain.

Classical Physics

Maxwell's Equations

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} == -\frac{d}{dt} \int_{Surface} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = \mu_0 (I_{through} + \varepsilon_0 \frac{d}{dt} \int_{surface} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}})$$

E, B →

← Q_{in}, I_{thr}

Newton's Laws

$$m \frac{d}{dt} \vec{\mathbf{v}}_{\mathbf{i}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}}_{\mathbf{i}} \times \vec{\mathbf{B}}\right)$$

$$\frac{d}{dt} \vec{\mathbf{x}}_{\mathbf{i}} = \vec{\mathbf{v}}_{\mathbf{i}}$$

$$Q_{in} = \sum_i q_i$$

$$I_{through} \vec{\mathbf{L}} = \sum_i q_i \vec{\mathbf{v}}_{\mathbf{i}}$$

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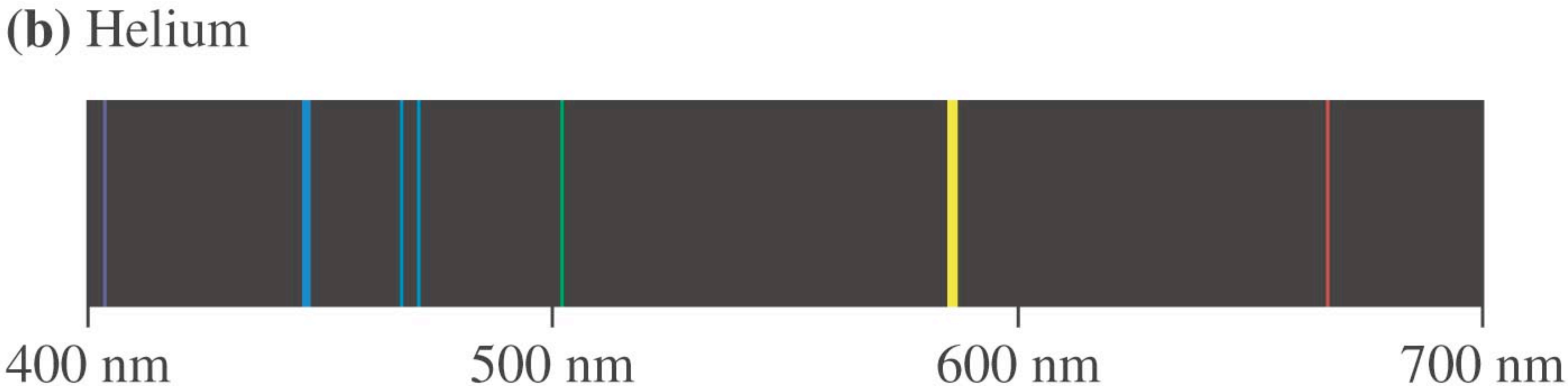
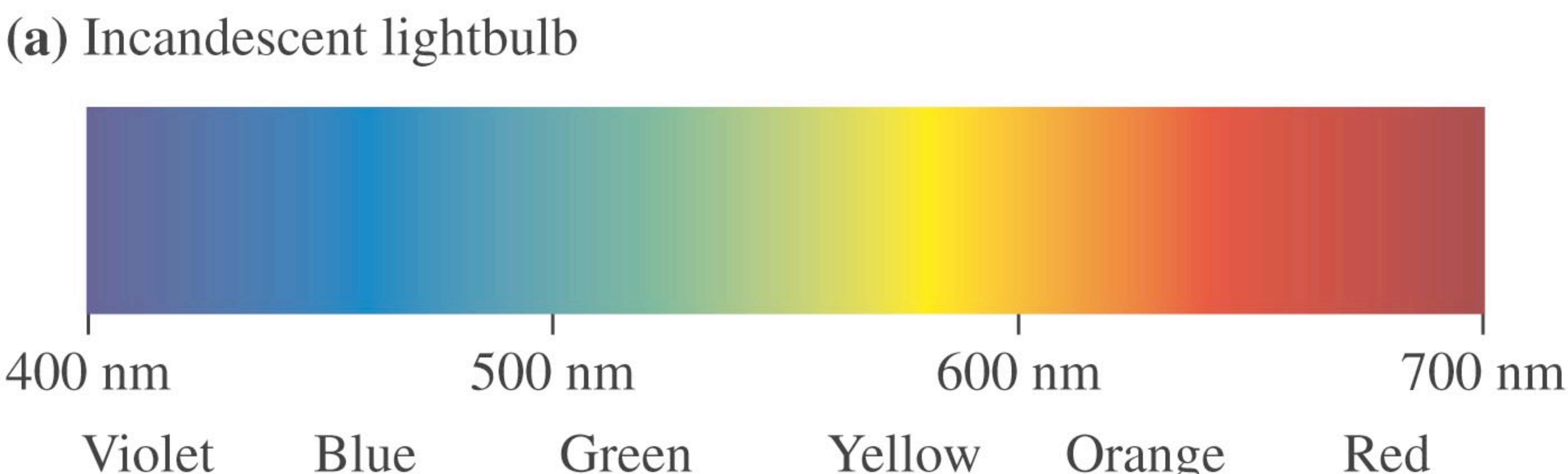
Discrete spectra of atoms - pass light through a diffraction grating

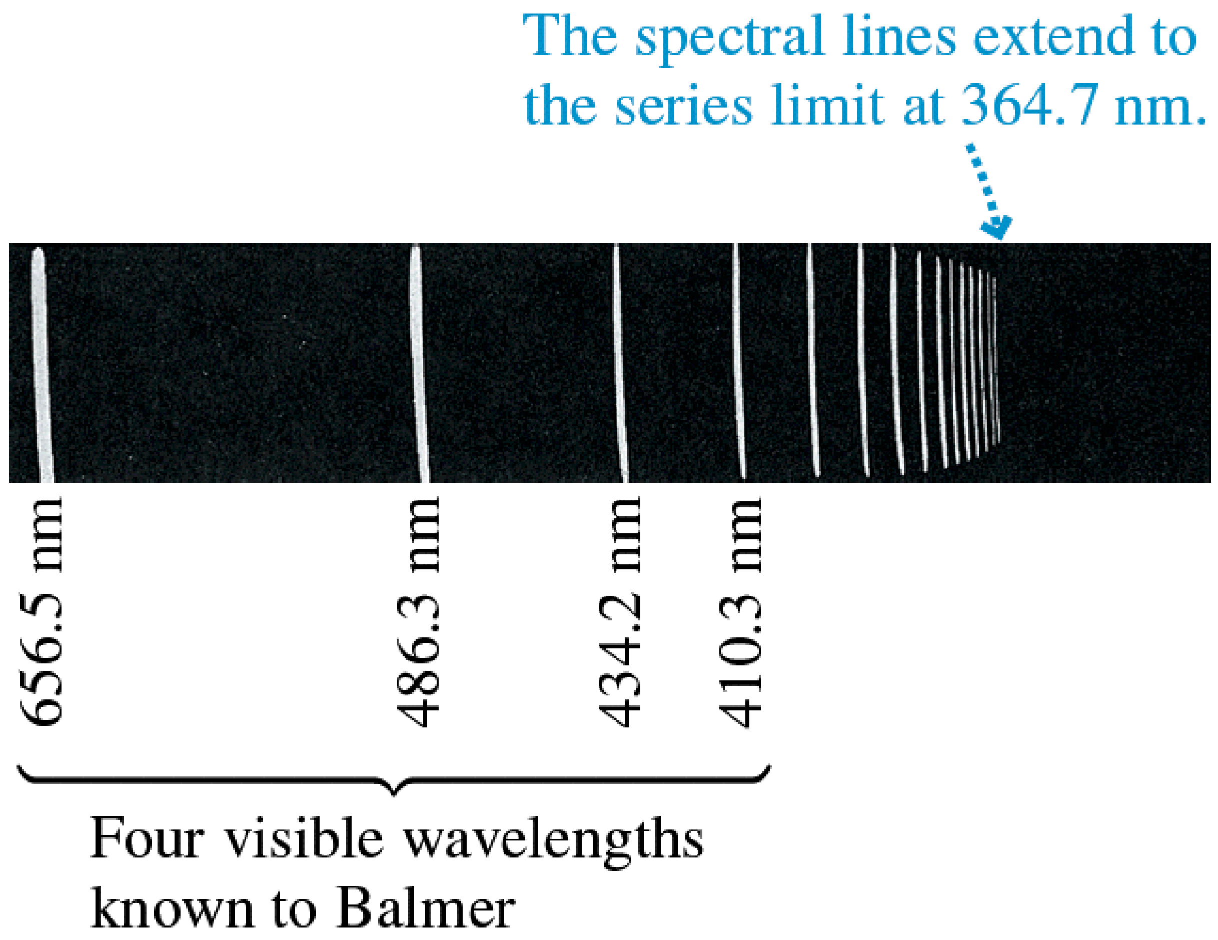




(c) Mercury

(d) Neon

FIGURE 25.3 The Balmer series of hydrogen as seen on the photographic plate of a spectrometer.



The Spectrum of Hydrogen

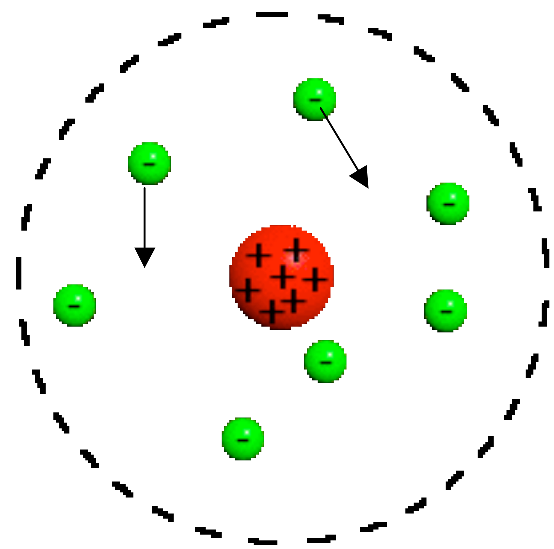
- Hydrogen is the simplest atom, with one electron orbiting a proton, and it also has the simplest atomic spectrum.
- The emission lines have wavelengths which correspond to two integers, *m* and *n*.
- Every line in the hydrogen spectrum has a wavelength given by

$$\lambda = \frac{91.18\text{ nm}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \qquad \begin{cases} m = 1 & \text{Lyman series} \\ m = 2 & \text{Balmer series} \\ m = 3 & \text{Paschen series} \\ \vdots & \end{cases}$$

$$n = m + 1, m + 2, \ldots$$

A classical picture of radiation emission from an atom

Electrons move around the nucleus. (Proposed by Rutherford as a result of experiments)



This creates an oscillating current density.

The oscillating current radiates like an antenna. This is the light that the atom emits.

Problems: This model predicts a continuous spectrum.
If you calculate the rate at which energy is radiated, all electrons would quickly spiral into the nucleus. Neither is observed.

The problems are fixed by

Abandoning Newton's Laws and replacing the description of electron motion by something that has wave aspects.

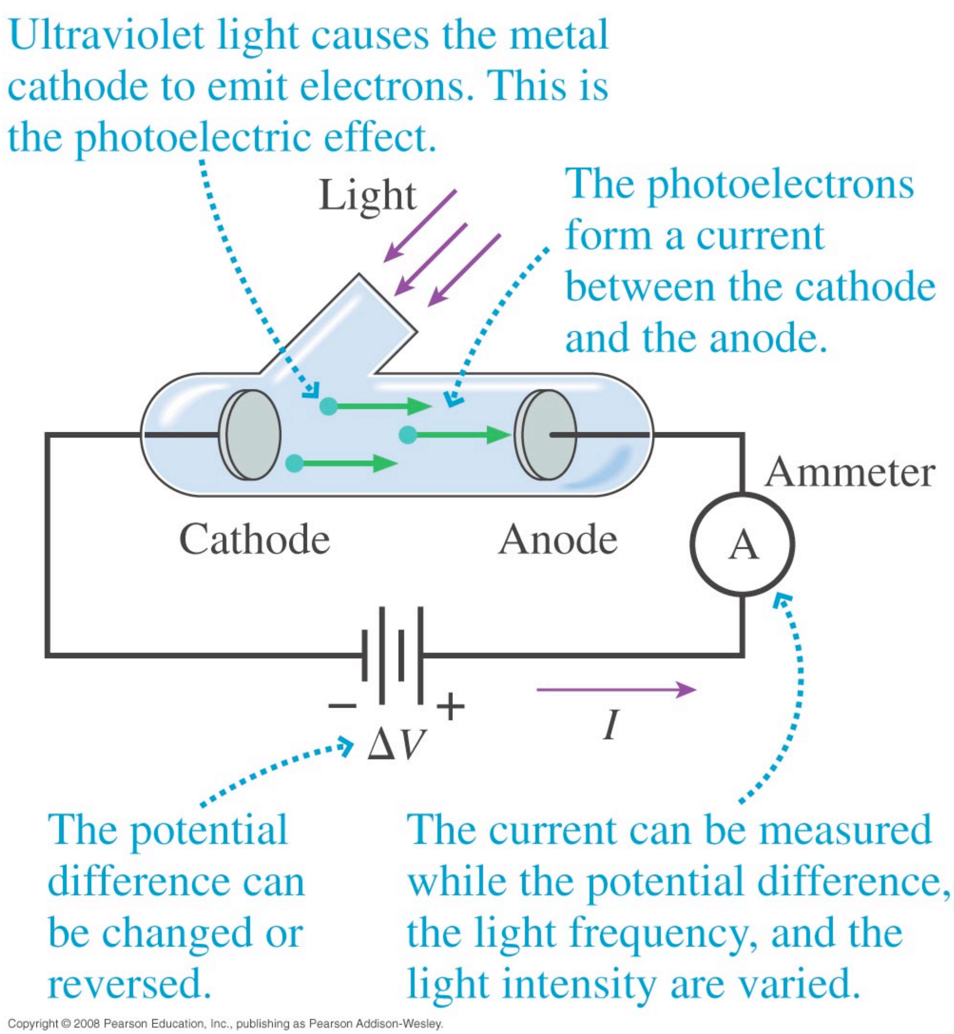
Waves in a confined region of space can only have certain frequencies.

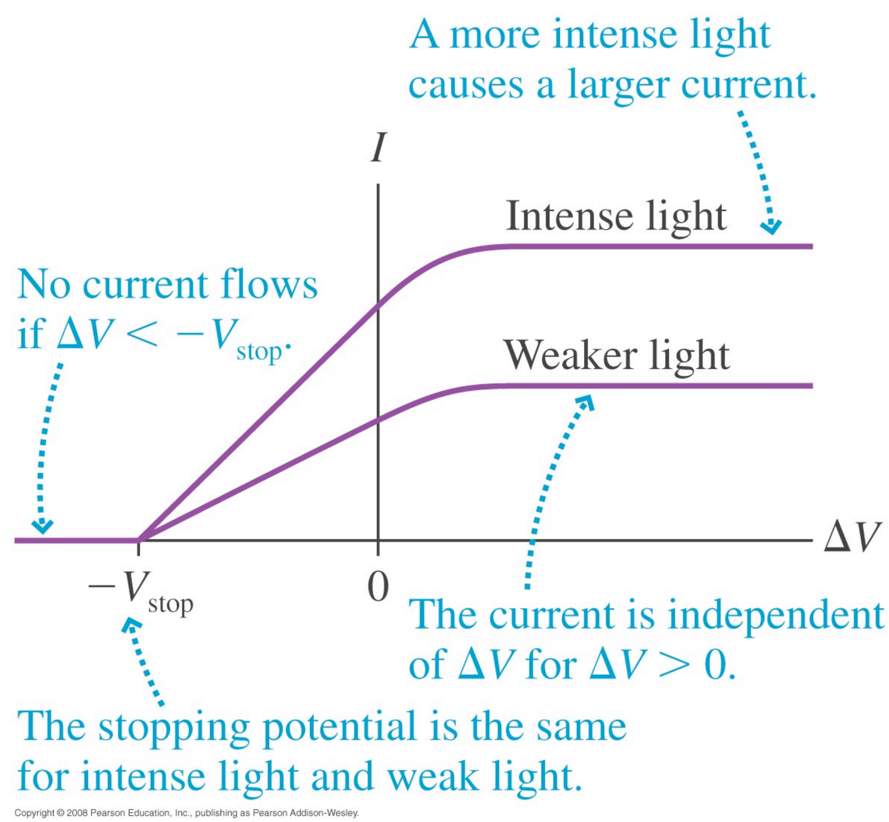
These can be related to the allowed energies of an electron in an atom.

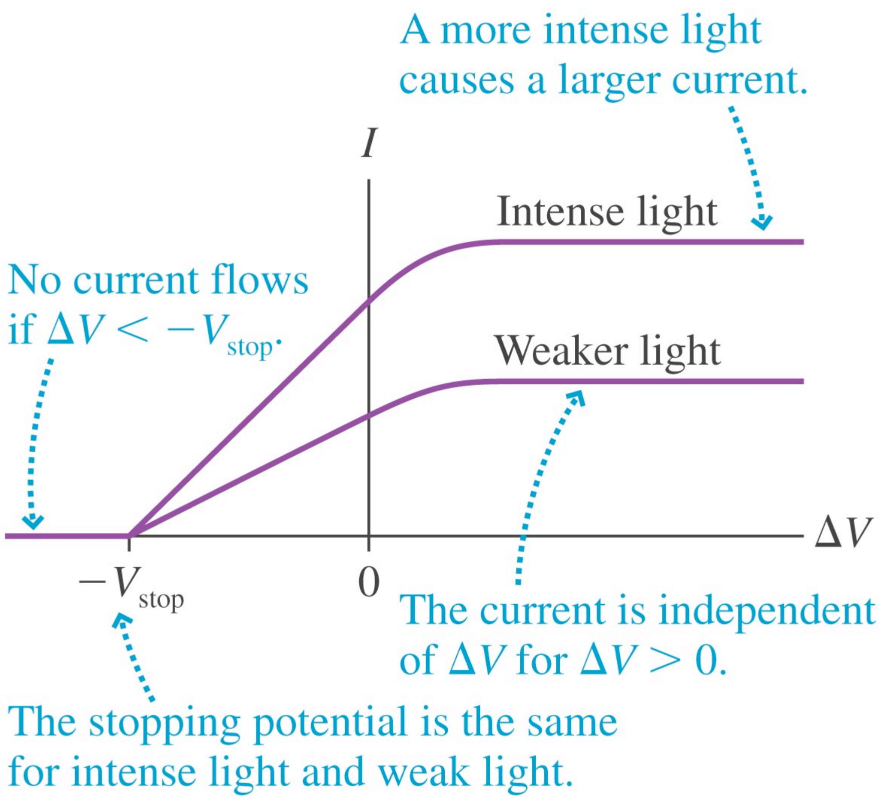
Results in discrete spectra. Explains Observations.

Light appears to come in chunks - particle like

Made clear by Einstein's explanation of the photoelectric effect.







V_{stop} depends on frequency of light

Planck's Constant

$$h = 6.63 \times 10^{-34} \text{ Joule-seconds}$$

Einstein showed that this behavior could be explained if:

1. Light consists of discrete massless units called photons.
2. Each Photon has energy $E_{photon} = hf$.
3. Many photons together act like the continuous field predicted by Maxwell's equations.

EXAMPLE 25.2 The energy of a photon

QUESTIONS:

EXAMPLE 25.2 The energy of a photon

550 nm is the average wavelength of visible light.

a. What is the energy of a photon with a wavelength of 550 nm?
b. A typical incandescent lightbulb emits about 1 J of visible light energy every second. Estimate the number of photons emitted per second.

EXAMPLE 25.2 The energy of a photon

SOLVE a. The frequency of the photon is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\ \text{m/s}}{550 \times 10^{-9}\ \text{m}} = 5.4 \times 10^{14}\ \text{Hz}$$

Equation 25.4 gives us the energy of this photon:

$$E_{\text{photon}} = hf = (6.63 \times 10^{-34}\ \text{J s})(5.4 \times 10^{14}\ \text{Hz})$$
$$= 3.6 \times 10^{-19}\ \text{J}$$

This is an extremely small energy!

EXAMPLE 25.2 The energy of a photon

b. The photons emitted by a lightbulb span a range of energies because the light spans a range of wavelengths, but the *average* photon energy corresponds to a wavelength near 550 nm. Thus we can estimate the number of photons in 1 J of light as

$$N \approx \frac{1\ \text{J}}{3.6 \times 10^{-19}\ \text{J/photon}} \approx 3 \times 10^{18}\ \text{photons}$$

A lightbulb emits about 3×10^{18} photons every second.

EXAMPLE 25.2 The energy of a photon

ASSESS This is a staggeringly large number. It’s not surprising that in our everyday life we would sense only the river and not the individual particles within the flow.

What is the energy associated with the shortest wavelength transition in H?

$$\lambda = \frac{91.18\ \text{nm}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \quad \begin{cases} m = 1 & \text{Lyman series} \\ m = 2 & \text{Balmer series} \\ m = 3 & \text{Paschen series} \\ \vdots & \end{cases}$$

$n = m + 1, m + 2, \ldots$

$$f = c / \lambda$$

$$f_{mn} = (c / 91.18\text{nm})\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

Take $m = 1, \quad n = \infty$ $\qquad f_{1,\infty} = 3.29 \times 10^{15}\,\text{Hz}$

Energy $E_{1,\infty} = hf_{1,\infty} = \left(6.63 \times 10^{-34}\right)\left(3.29 \times 10^{15}\right) = 2.18 \times 10^{-18}\,\text{J}$

What is the potential energy associated with the electron and proton in an H?

$$U = \frac{-e^2}{4\pi\varepsilon_0 r}$$

$e = 1.6 \times 10^{-19}$ C

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$\text{r} \sim \text{a}_0 = 5.3 \times 10^{-11}$ m

$U = 4.34 \times 10^{-18}$ J

Discrete Spectra is a consequence of electrons radiating individual photons

Does a photon of red light have more energy or less energy than a photon of blue light?

A. More energy
B. Less energy

X-Ray Diffraction

The figure shows a simple cubic lattice of atoms. The crystal structure of most materials is more complex than this, but a cubic lattice will help you understand the ideas of x-ray diffraction.

FIGURE 25.5 Atoms arranged in a cubic lattice.

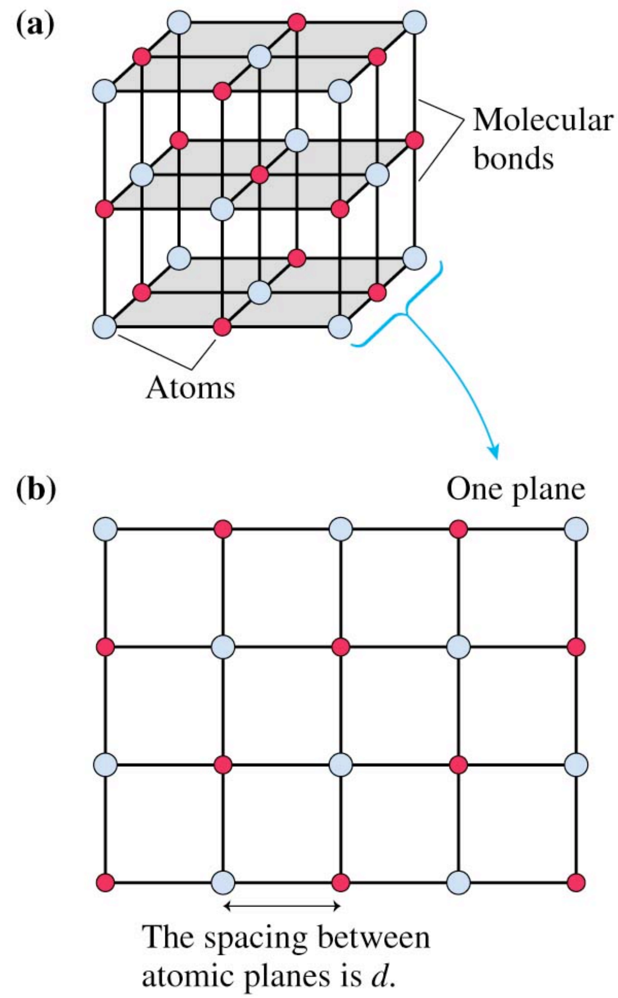
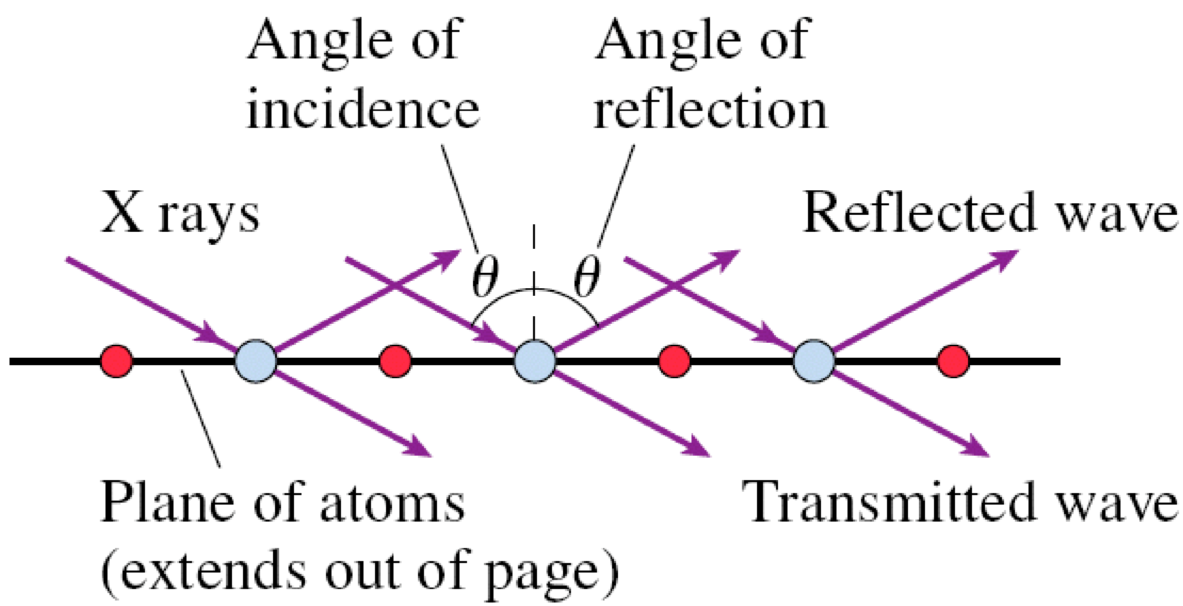


FIGURE 25.6 The x-ray reflections from parallel atomic planes interfere constructively to cause strong reflections for certain angles of incidence.

(a) X rays are transmitted and reflected at one plane of atoms.



In this case (one layer) reflection is weak

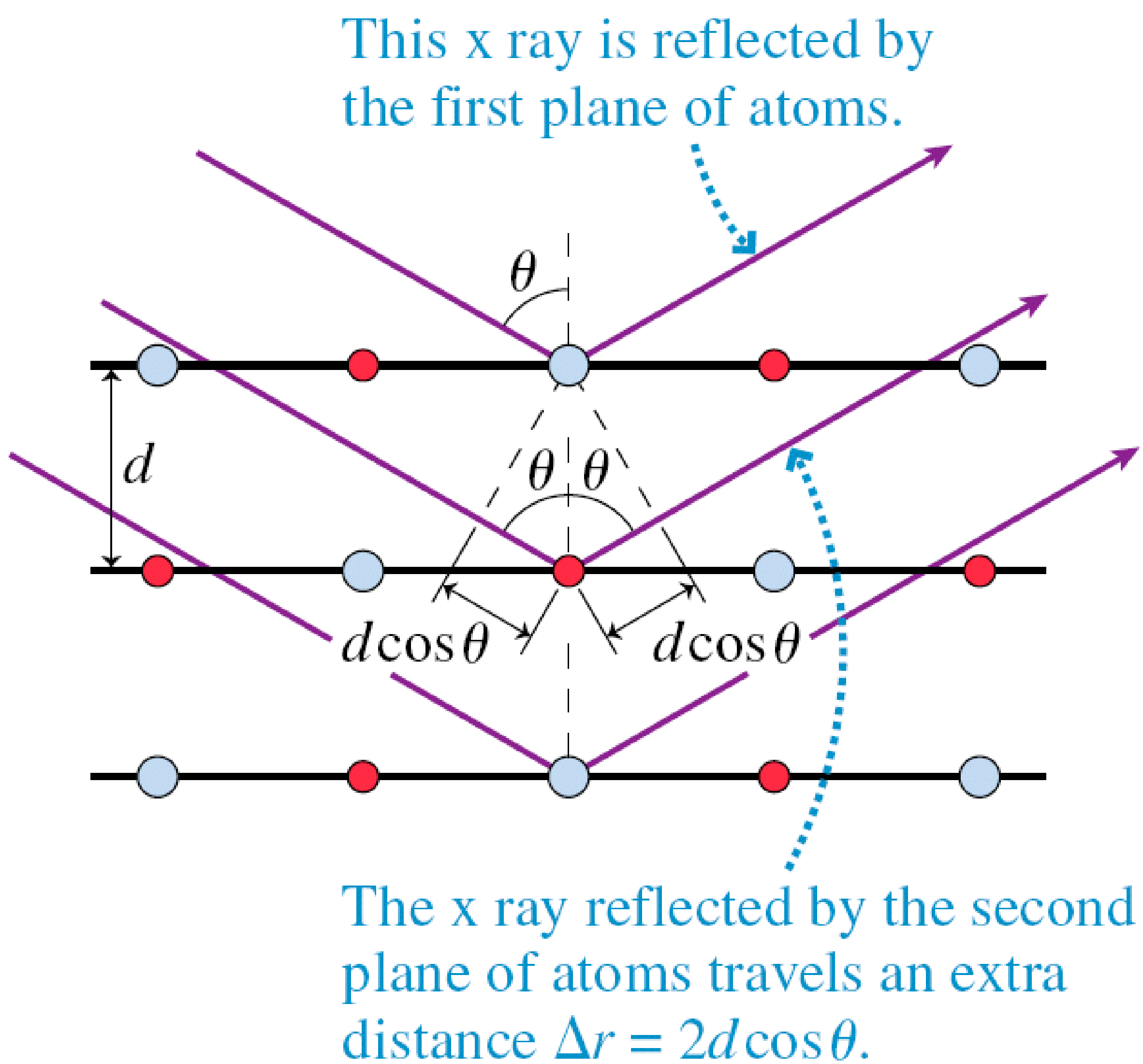
When many layers are present reflection can be strong, but only when constructive interference occurs.

Difference in path length for constructive interference

$$\Delta r = 2d\cos\theta_m = m\lambda$$

Bragg Condition

(b) The reflections from parallel planes interfere.



X-Ray Diffraction

- The wave reflecting from any particular plane travels an extra distance $\Delta r = 2d\cos\theta$ before combining with the reflection from the plane immediately above it, where d is the spacing between the atomic planes.
- If $\Delta r = m\lambda$, these two waves will be in phase when they recombine.
- Consequently, x rays will strongly reflect from the crystal when the angle of incidence θ_m satisfies

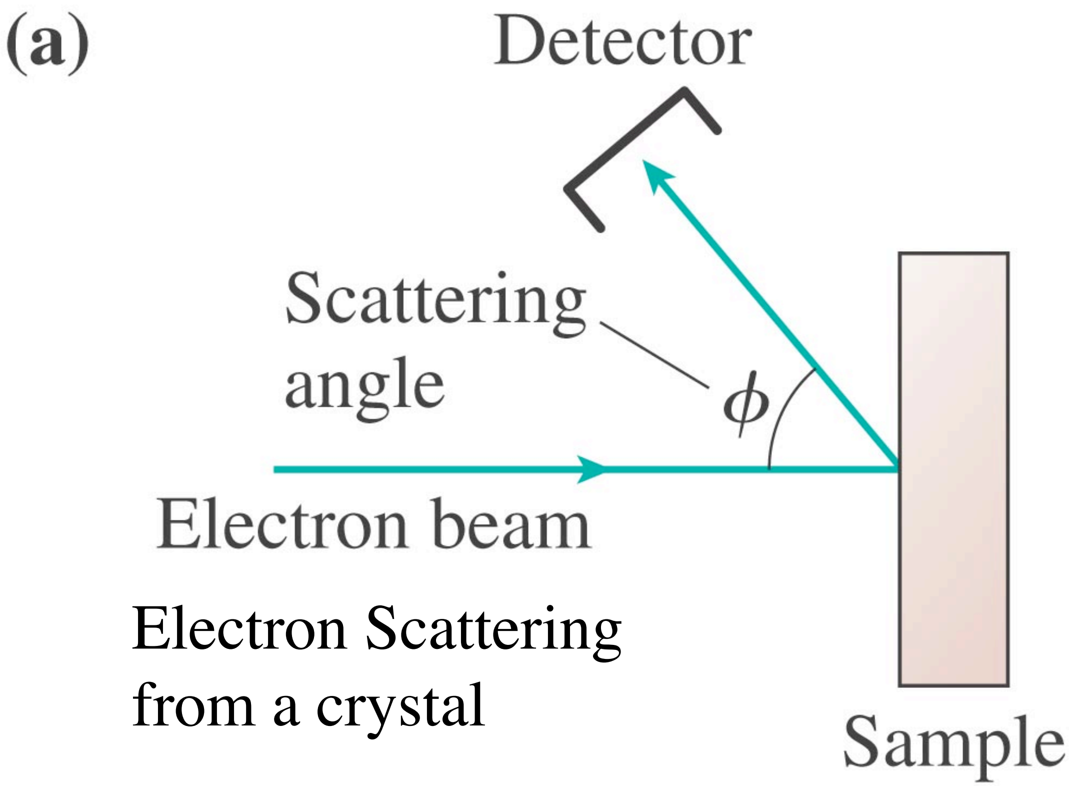
$$\Delta r = 2d\cos\theta_m = m\lambda \qquad m = 1, 2, 3, \ldots$$

Equation 25.3 is called the Bragg condition.

Matter Waves

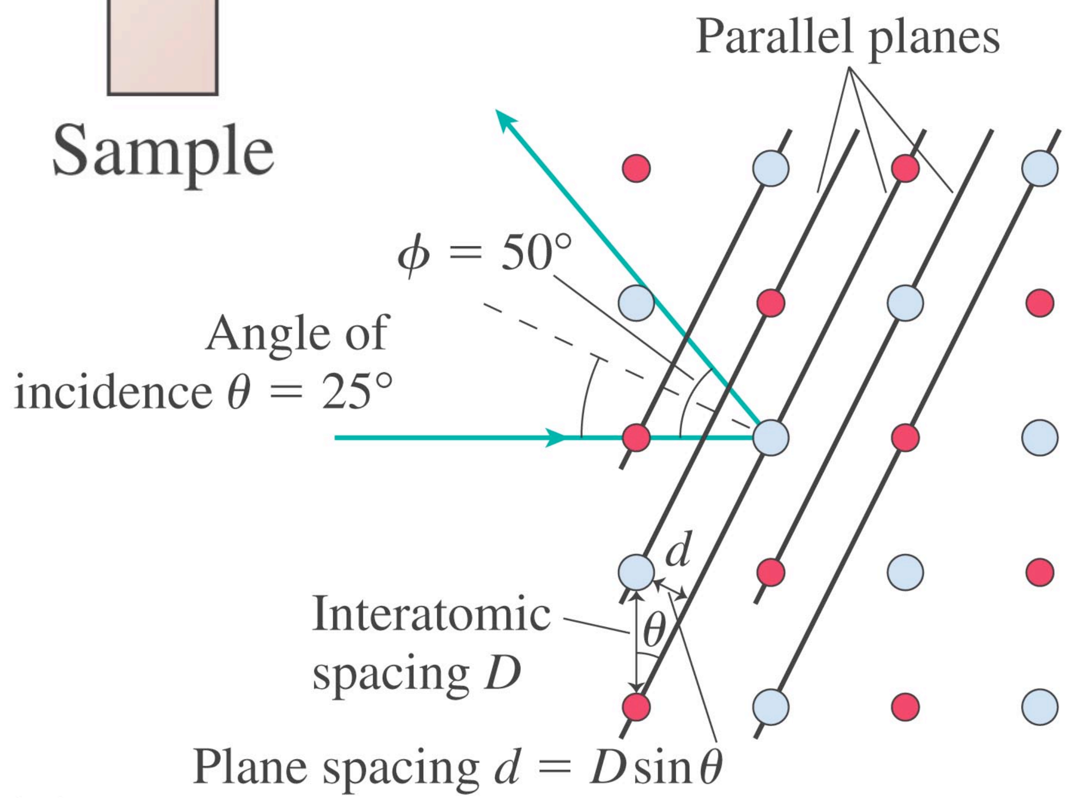
- In 1927 Davisson and Germer were studying how electrons scatter from the surface of metals.
- They found that electrons incident normal to the crystal face at a speed of 4.35×10^6 m/s scattered at $\phi = 50°$.
- This scattering can be interpreted as a mirror-like reflection from the atomic planes that slice diagonally through the crystal.
- The angle of incidence on this set of planes is the angle θ_m in $2d \cos \theta_m = m\lambda$, the Bragg condition for diffraction.
- Davisson and Germer found that the "electron wavelength" was

$$\lambda = D\sin(2\theta) = 0.165 \text{ nm}$$



Electron Scattering from a crystal

According to classical physics one would expect the detected signal to depend smoothly on angle.
Instead peaks were observed at specific angles.



Angle of incidence on plane θ

Separation of planes $d = D\sin\theta$

Strong reflection when

$$2d\cos\theta = \lambda$$

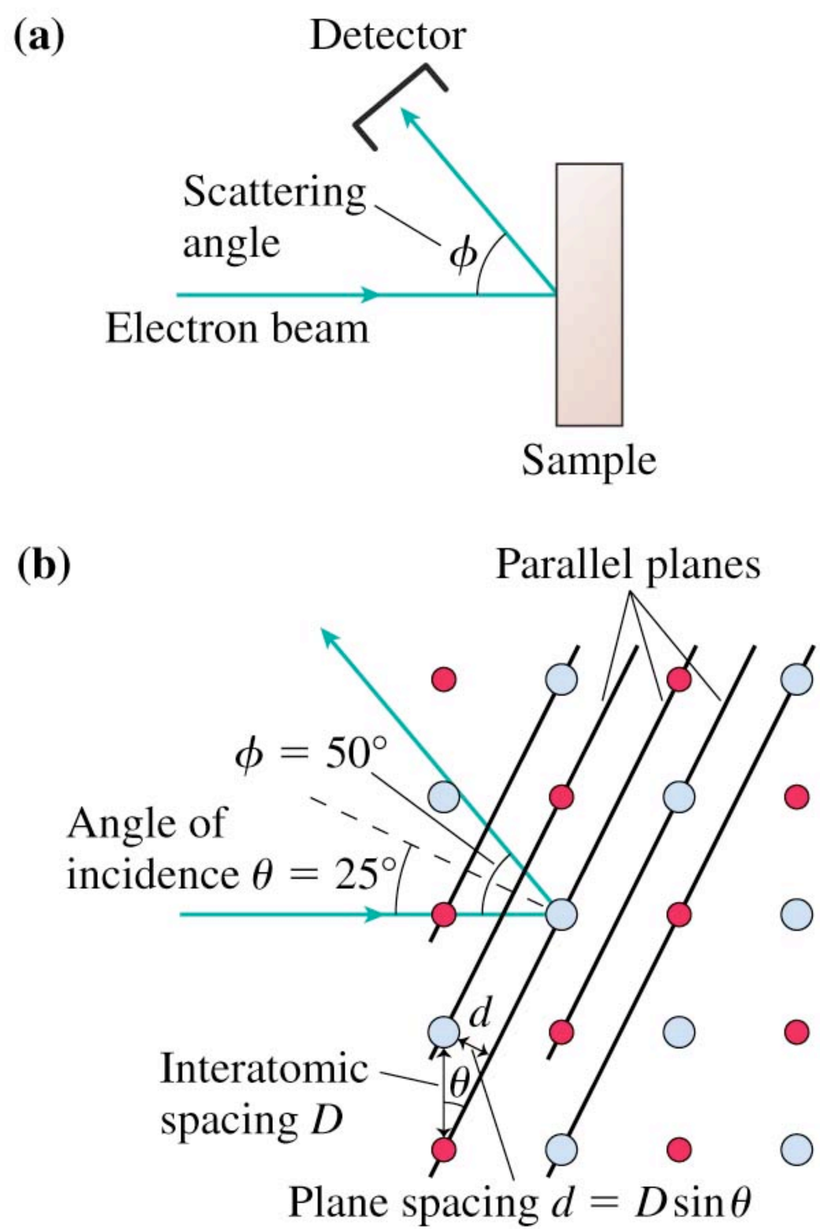
(m=1 Bragg condition)

Combining

$$2D\sin\theta\cos\theta = D\sin 2\theta = \lambda$$

$$\lambda = D\sin(2\theta) = 0.165 \text{ nm}$$

FIGURE 25.11 The Davisson-Germer experiment to study electrons scattered from metal surfaces.



The de Broglie Wavelength

De Broglie postulated that a particle of mass *m* and momentum $p = mv$ has a wavelength

$$\lambda = \frac{h}{p}$$

where *h* is Planck's constant. This wavelength for material particles is now called the **de Broglie wavelength.** It depends *inversely* on the particle's momentum, so the largest wave effects will occur for particles having the smallest momentum.

EXAMPLE 25.4 The de Broglie wavelength of an electron

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Find the de Broglie wavelength of an electron with a speed of 4.35×10^6 m/s, the speed in the Davisson-Germer experiment.

EXAMPLE 25.4 The de Broglie wavelength of an electron

SOLVE The mass of an electron is 9.11×10^{-31} kg. Its de Broglie wavelength at this speed is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = 0.167 \text{ nm}$$

EXAMPLE 25.4 The de Broglie wavelength of an electron

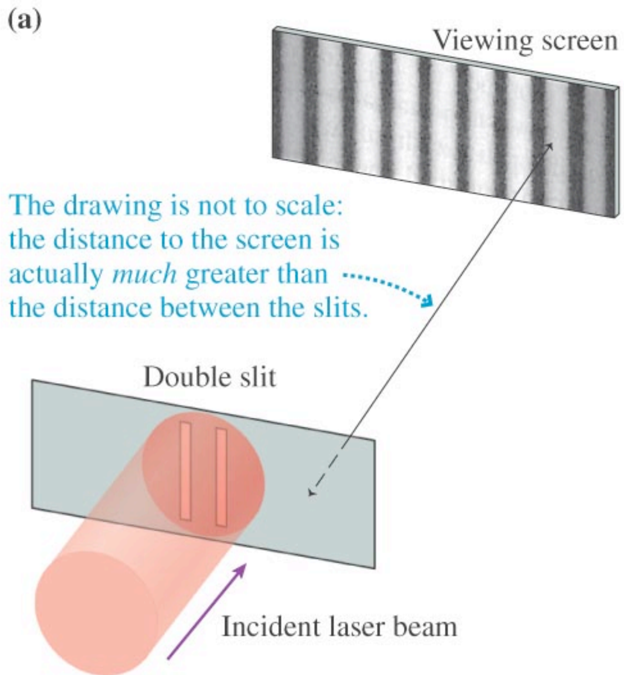
ASSESS This result is in near-perfect agreement with Davisson and Germer's experimentally determined wavelength of 0.165 nm! Electrons moving with speeds in this range have de Broglie wavelengths very similar to those of x rays. These wavelengths are exactly the right size to be diffracted by atomic crystals.

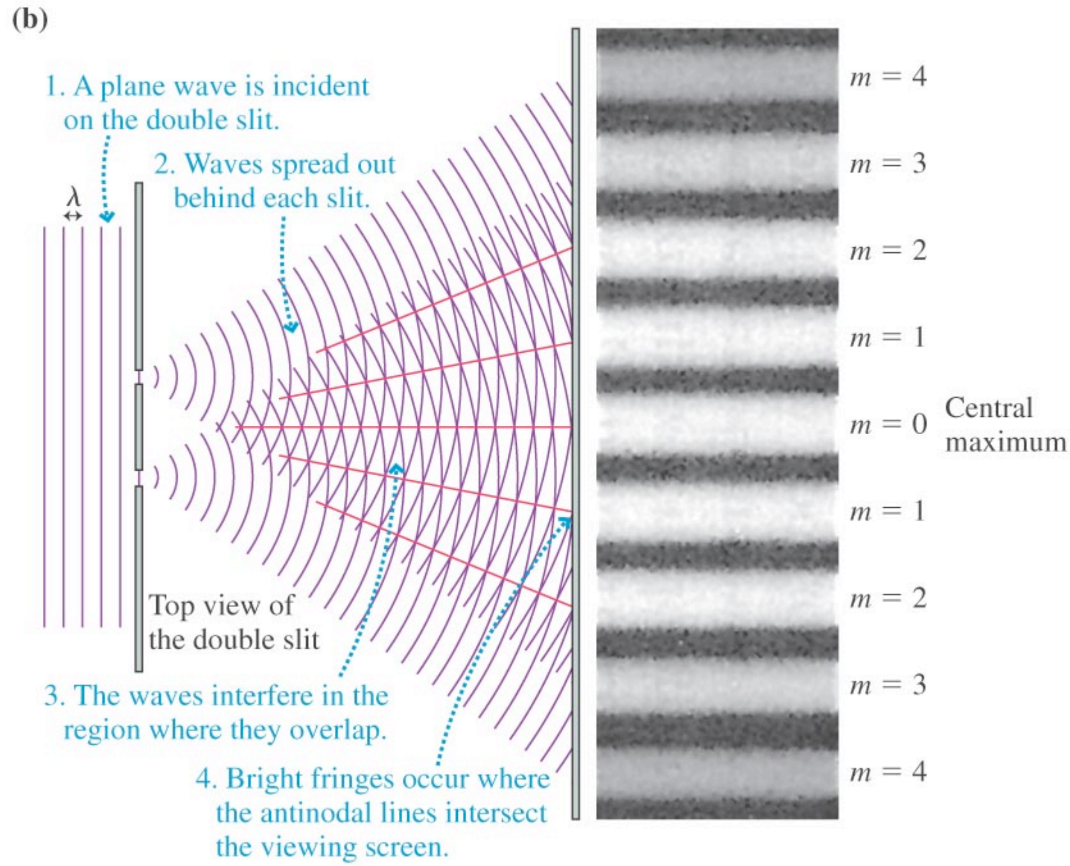
A proton, an electron and an oxygen atom each pass at the same speed through a 1-μm-wide slit. Which will produce a wider diffraction pattern on a detector behind the slit?

A. The oxygen atom.
B. The proton.
C. The electron.
D. All three will be the same.
E. None of them will produce a diffraction pattern.

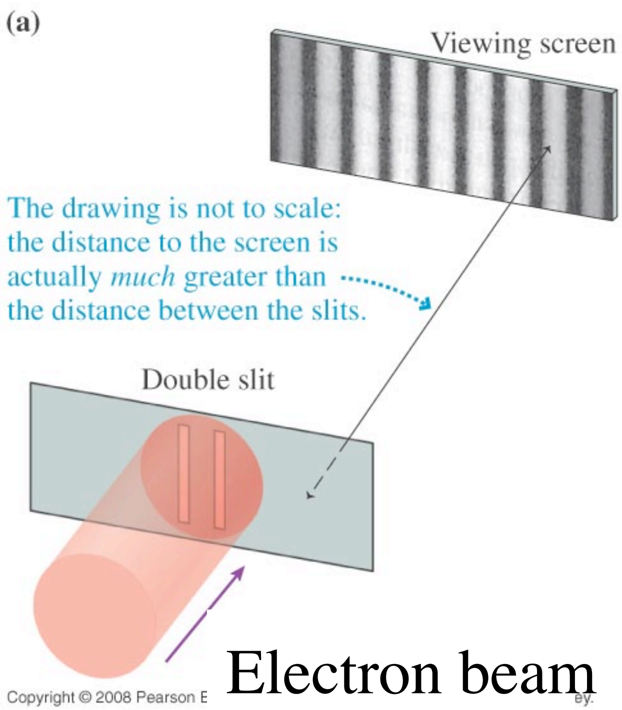
Interference of light
Double Slit experiment

Coherence because sources are at exactly the same frequency



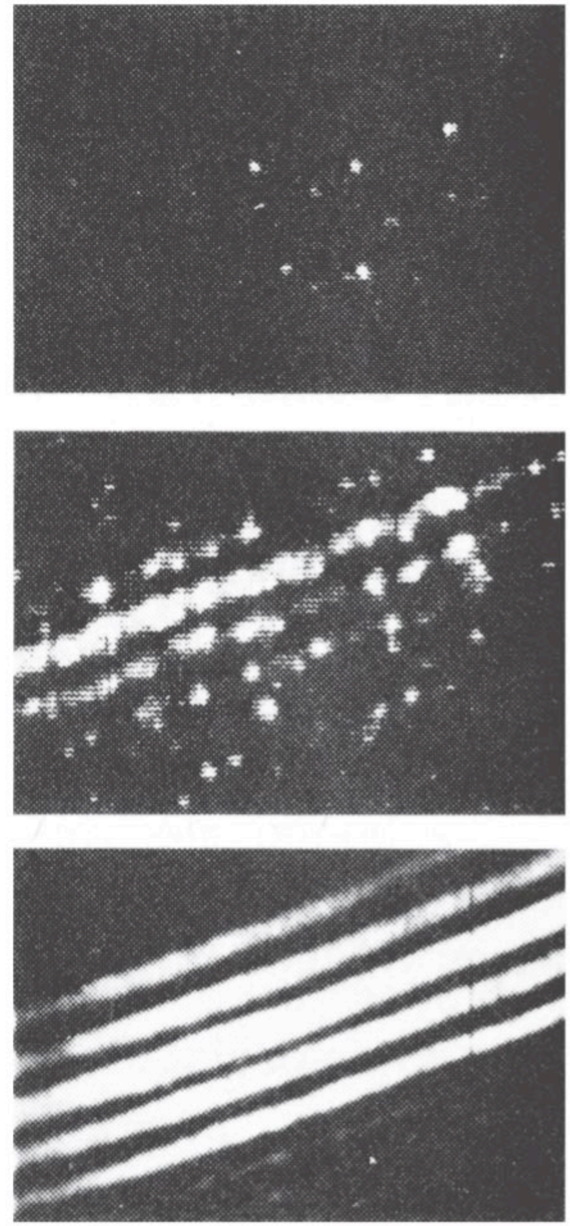


Diffraction of Matter



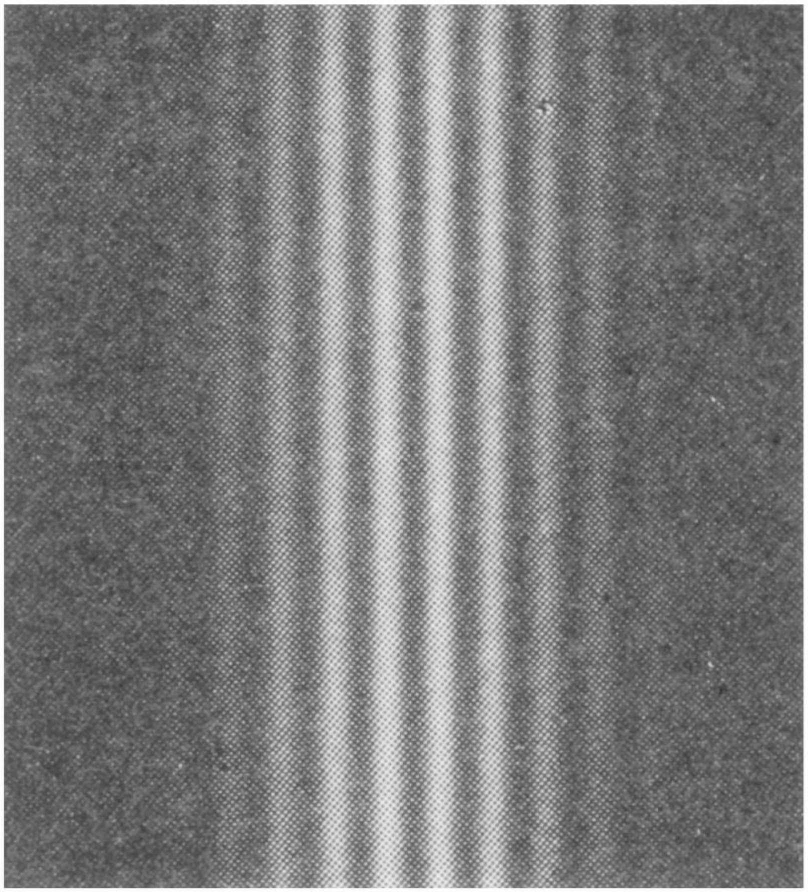
Electron beam

Electrons arrive one by one. Hitting the screen at discrete points. But over time a diffraction pattern is built up!



Puzzle: When it hits the screen it acts like a particle, but somehow it went through both slits.

(a) Double-slit interference of electrons



(b) Double-slit interference of neutrons

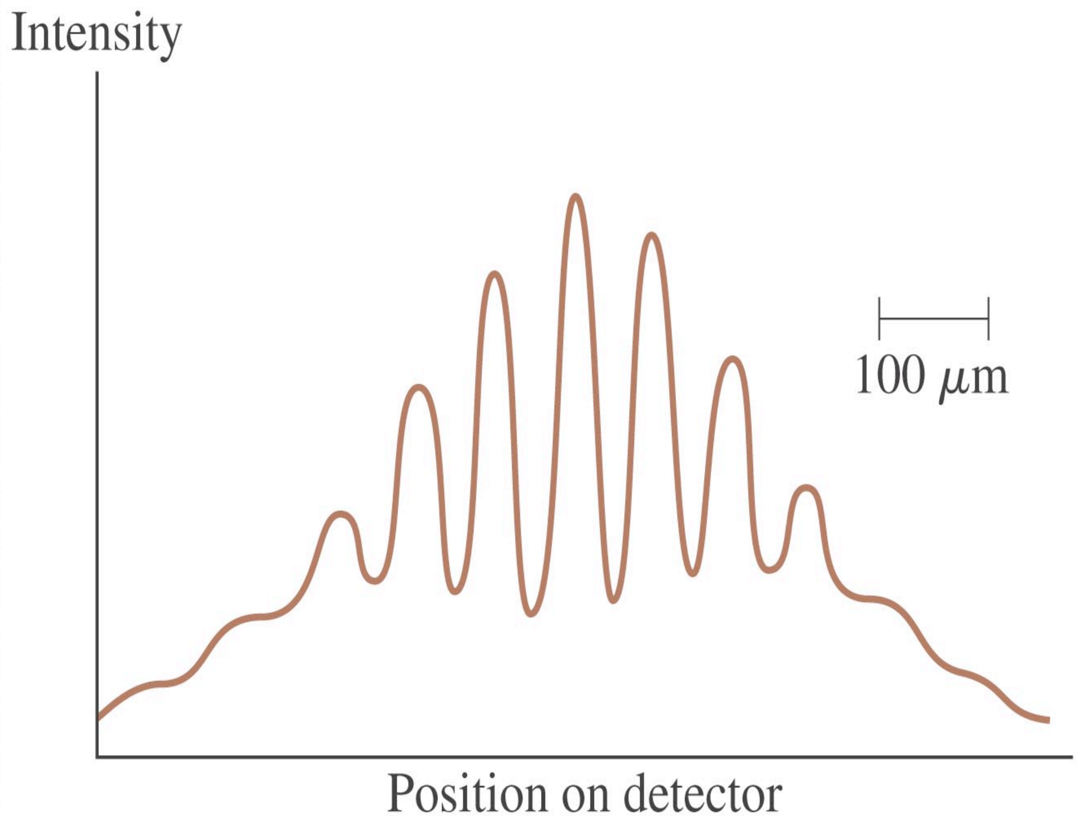
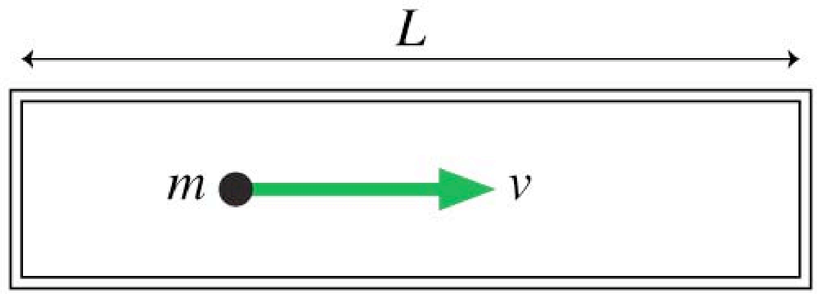
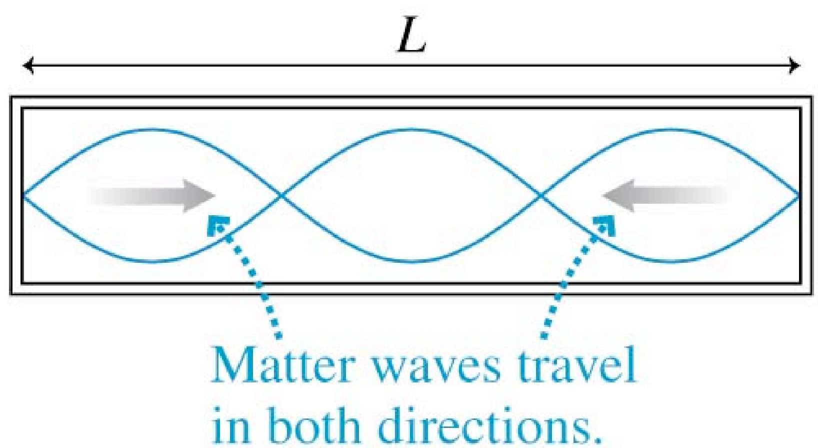


FIGURE 25.16 A particle of mass m confined in a box of length L.

(a) A classical particle of mass m bounces back and forth between the ends.



(b) Matter waves moving in opposite directions create standing waves.



Energy is Quantized

Relating wavelength to momentum by using the de Broglie equation, the discrete values of wavelength of the particle in the box lead to discrete values of momentum, and discrete levels of energy

$$E_n = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2}{8mL^2}n^2 \qquad n = 1, 2, 3, 4, \ldots$$

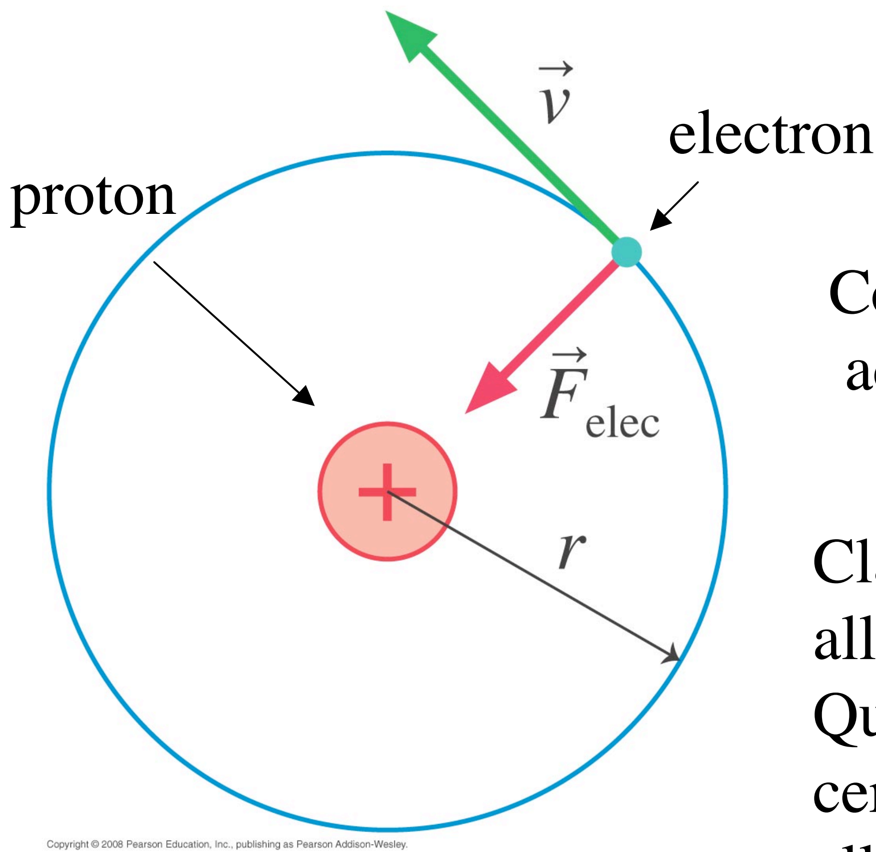
A confined particle can only have certain energies. This is called the **quantization** of energy. The number *n* is called the **quantum number**; each value of *n* characterizes one **energy level** of the particle in the box.

A proton, an electron and an oxygen atom are each confined in a 1-nm-long box. Rank in order, from largest to smallest, the minimum possible energies of these particles.

A. $E_{\mathrm{O}} > E_{\mathrm{C}} > E_{\mathrm{H}}$
B. $E_{\mathrm{H}} > E_{\mathrm{C}} > E_{\mathrm{O}}$
C. $E_{\mathrm{O}} > E_{\mathrm{H}} > E_{\mathrm{C}}$
D. $E_{\mathrm{C}} > E_{\mathrm{O}} > E_{\mathrm{H}}$
E. $E_{\mathrm{H}} > E_{\mathrm{O}} > E_{\mathrm{C}}$

39. 6 Bohr Model of the Hydrogen Atom (Approximate QM treatment)

Classical Picture



$$m\vec{\mathbf{a}} = \vec{\mathbf{F}}$$

$$m\frac{v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

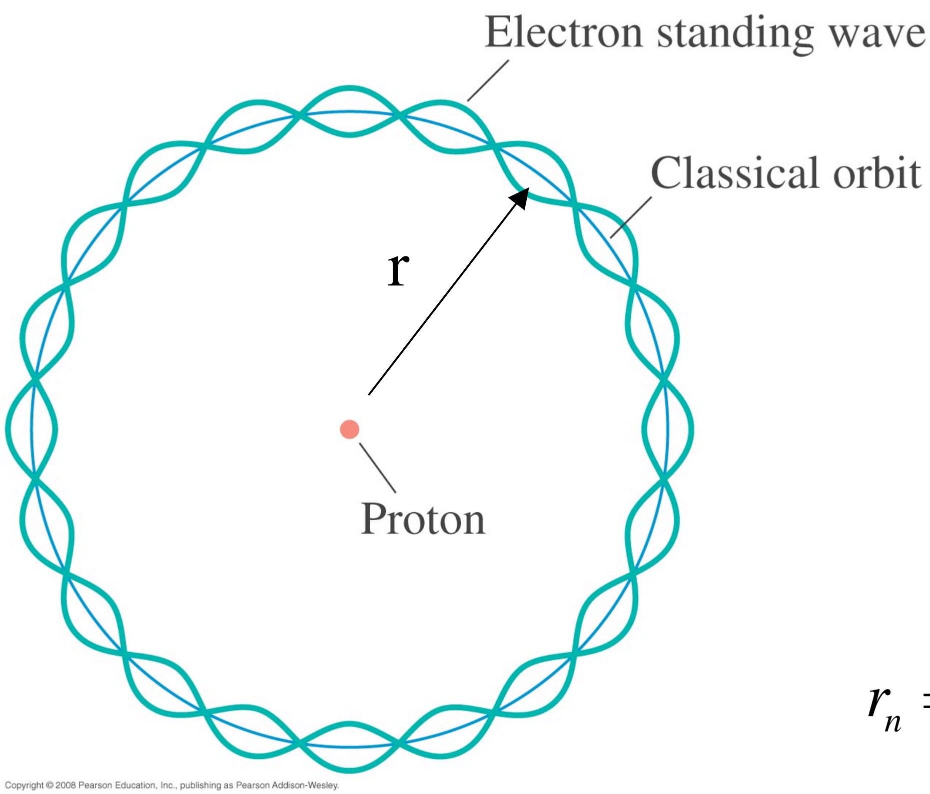
Centripetal acceleration

Coulomb force

Classically, any value of v and r are allowed so long as F=ma above. Quantum mechanics says only certain values of r and v are allowed.

Quantum mechanics: Orbit must be an integer # of de Broglie wavelengths

$$2\pi r = n\lambda$$



$$mv = h/\lambda = \frac{hn}{2\pi r}$$

Plug in and solve for r.

$$m\frac{v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

Only certain r's are allowed.

$$r_n = n^2 a_0 \qquad a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$$

Bohr radius $a_0 = 5.3\times 10^{-11}\,\mathrm{m}$

What are the total energies (Kinetic + Potential) of these states?

Kinetic Energy: $$K = m\frac{v_n^{\ 2}}{2}$$ ← use $$mv_n = h / \lambda_n = \frac{hn}{2\pi r_n}$$

Potential Energy: $$U = -\frac{e^2}{4\pi\varepsilon_0 r_n}$$

Combining, E=K+U $$E_n = -\frac{1}{n^2}\frac{e^2}{8\pi\varepsilon_0 a_0} = -\frac{2.18\times 10^{-18}}{n^2}\,\mathrm{J}$$

Gives Balmer spectrum

General Principles

The breakdown of classical physics

A closer look at light and matter finds that these classical models are not sufficient. Light and matter are neither particles nor waves, but exhibit characteristics of both.

Important Concepts

Light

- Exhibits interference and diffraction

  Wave-like: $c = \lambda f$

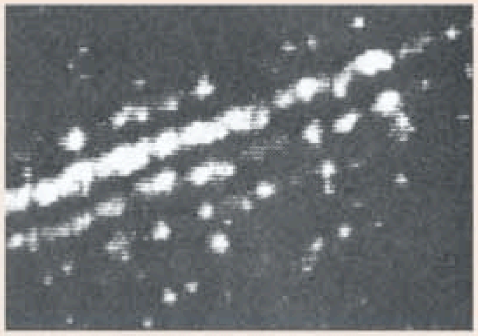
- Detected at localized positions

  Particle-like: $E = hf$

- Particle-like "chunks" of light are called photons.

Important Concepts

Matter



- Detected at localized positions

  Particle-like: $E = \frac{1}{2}mv^2$

- Exhibits interference and diffraction

  Wave-like: $\lambda = h/p$

- The wavelength is called the **de Broglie wavelength.**

Important Concepts

Quantization

A "particle" confined to a one-dimensional box of length L sets up a standing wave with the de Broglie wavelength. Because only certain wavelengths can oscillate, only certain discrete energies are allowed:

$$E_n = \frac{h^2}{8mL^2}n^2 \qquad n = 1, 2, 3, \ldots$$

Energy is quantized into discrete levels rather than being continuous as it is in classical physics. Quantization is not important for macroscopic objects, but energy quantization plays a very large role at the atomic level.