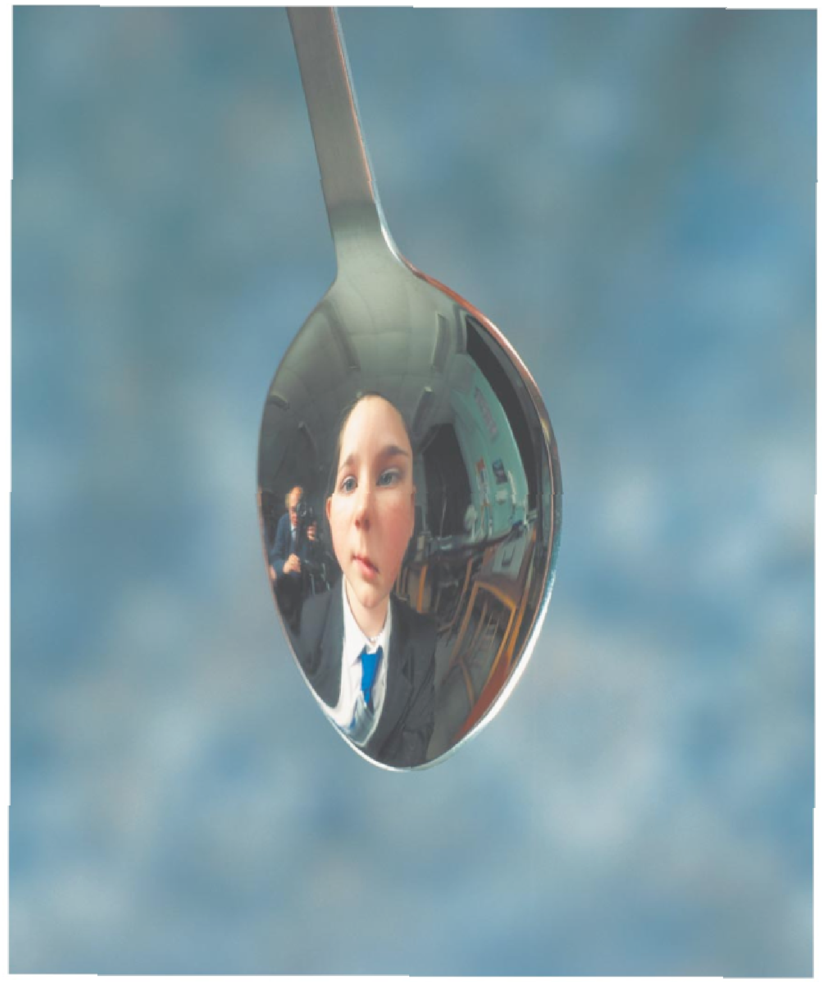
Chapter 23

Ray Optics

Why doesn't this work with real spoons?

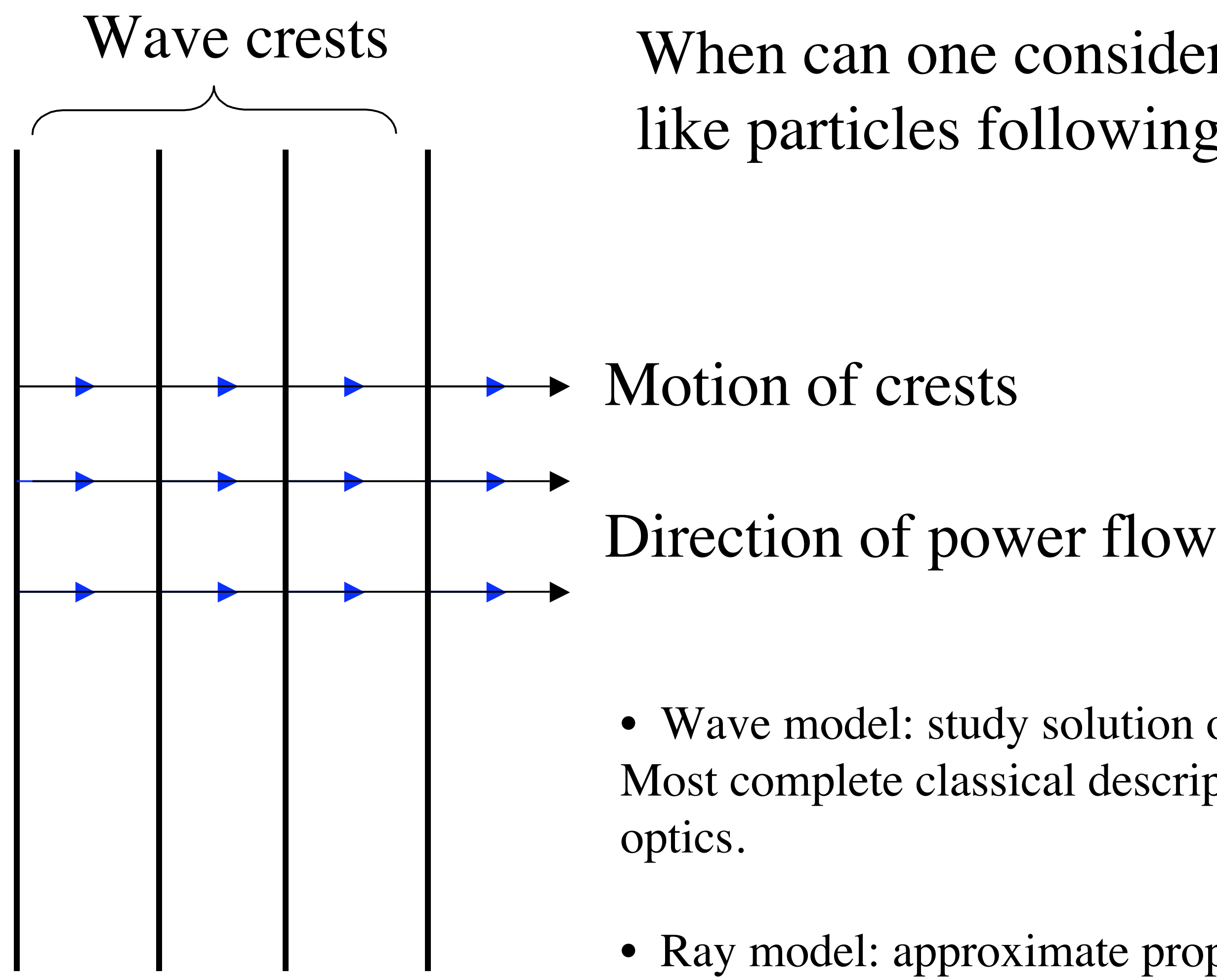


Chapter 23. Ray Optics

Topics:

- The Ray Model of Light
- Reflection
- Refraction
- Image Formation by Refraction
- Color and Dispersion
- Thin Lenses: Ray Tracing
- Thin Lenses: Refraction Theory
- Image Formation with Spherical Mirrors

When can one consider waves to be like particles following a trajectory?



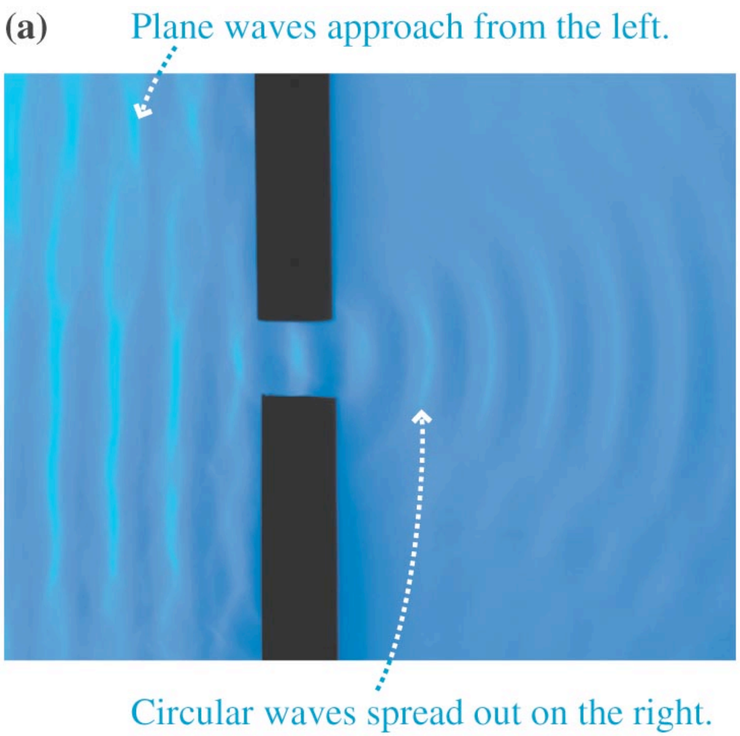
• Wave model: study solution of Maxwell equations. Most complete classical description. Called physical optics.

• Ray model: approximate propagation of light as that of particles following specific paths or “rays”. Called geometric optics.

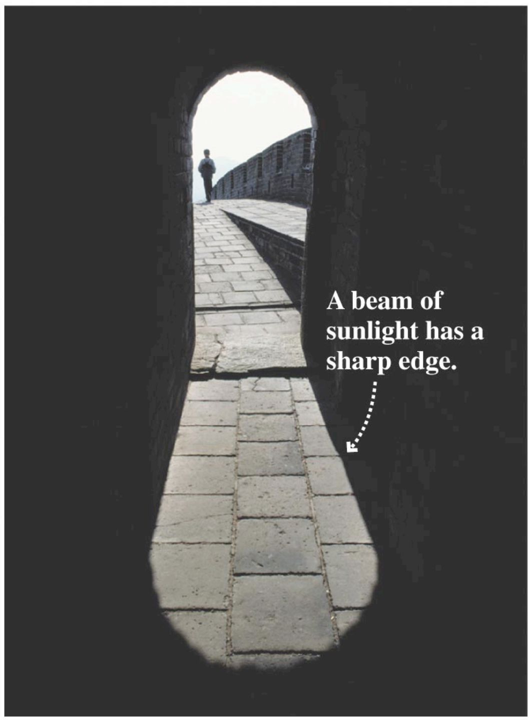
• Quantum optics: Light actually comes in chunks called photons

Wave Picture vs Ray Picture

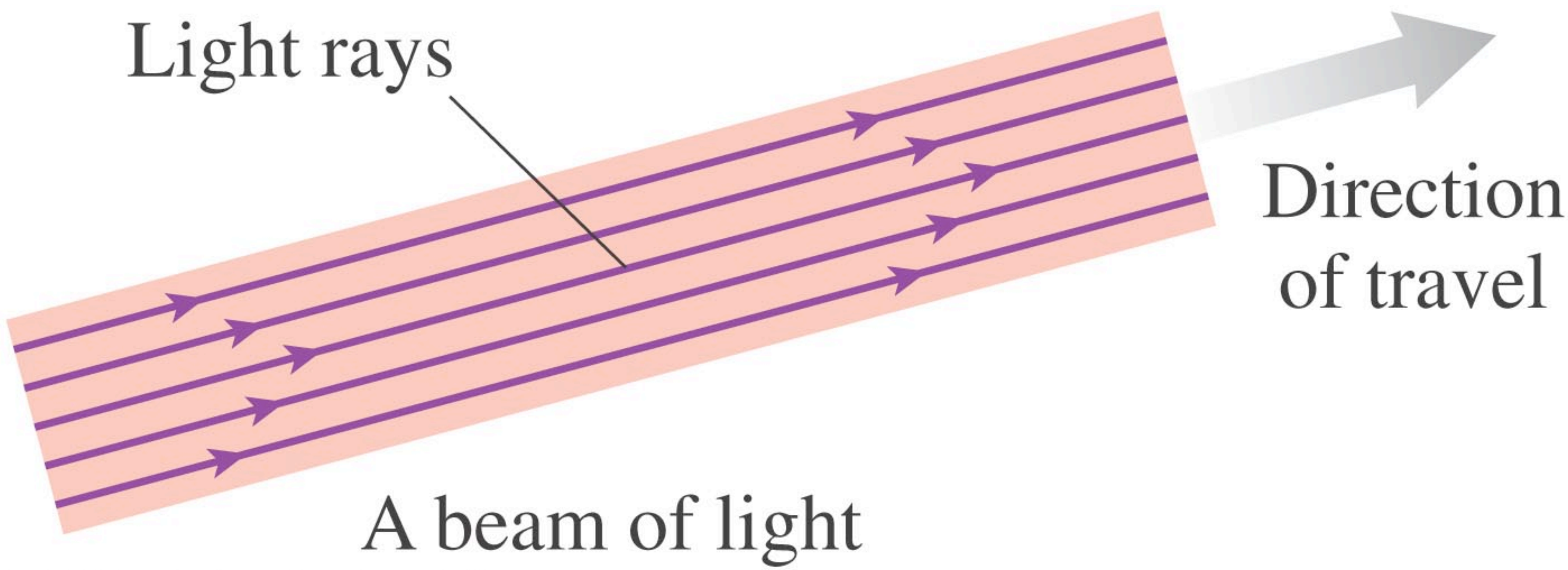
(a)

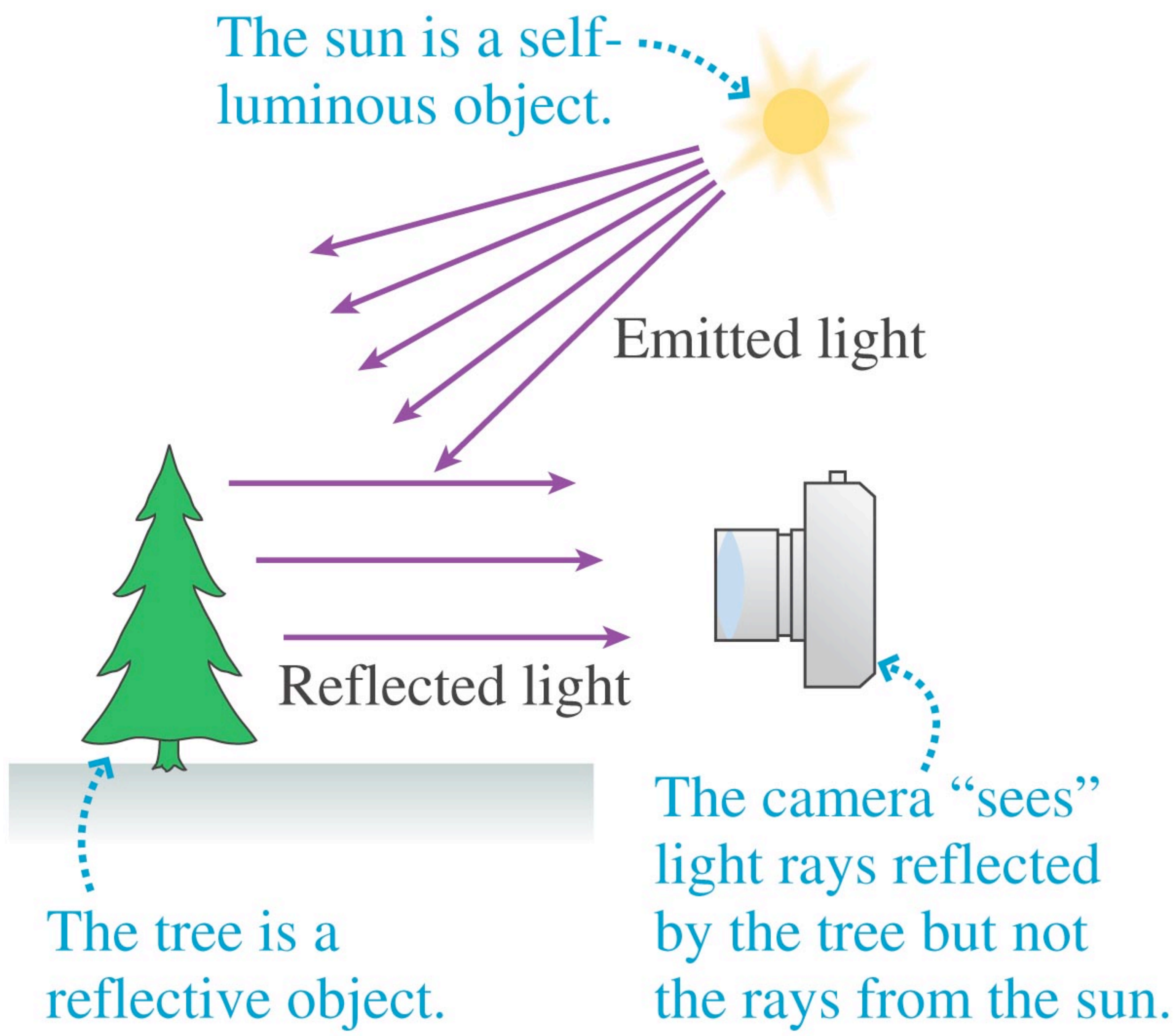


(b)

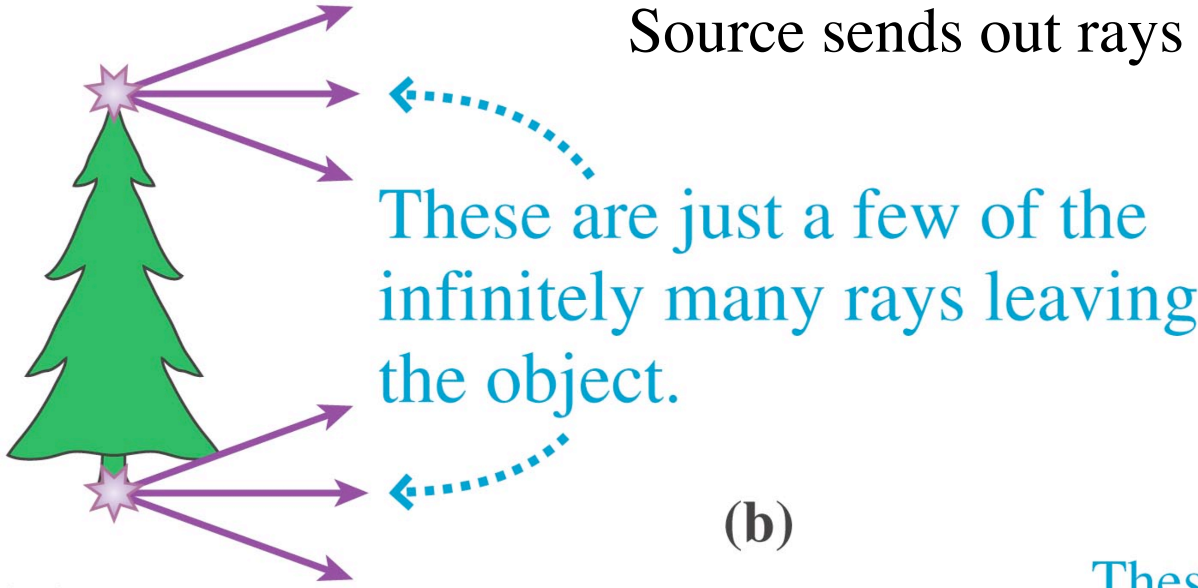


In the Ray Picture a beam of light is a bundle of parallel traveling rays



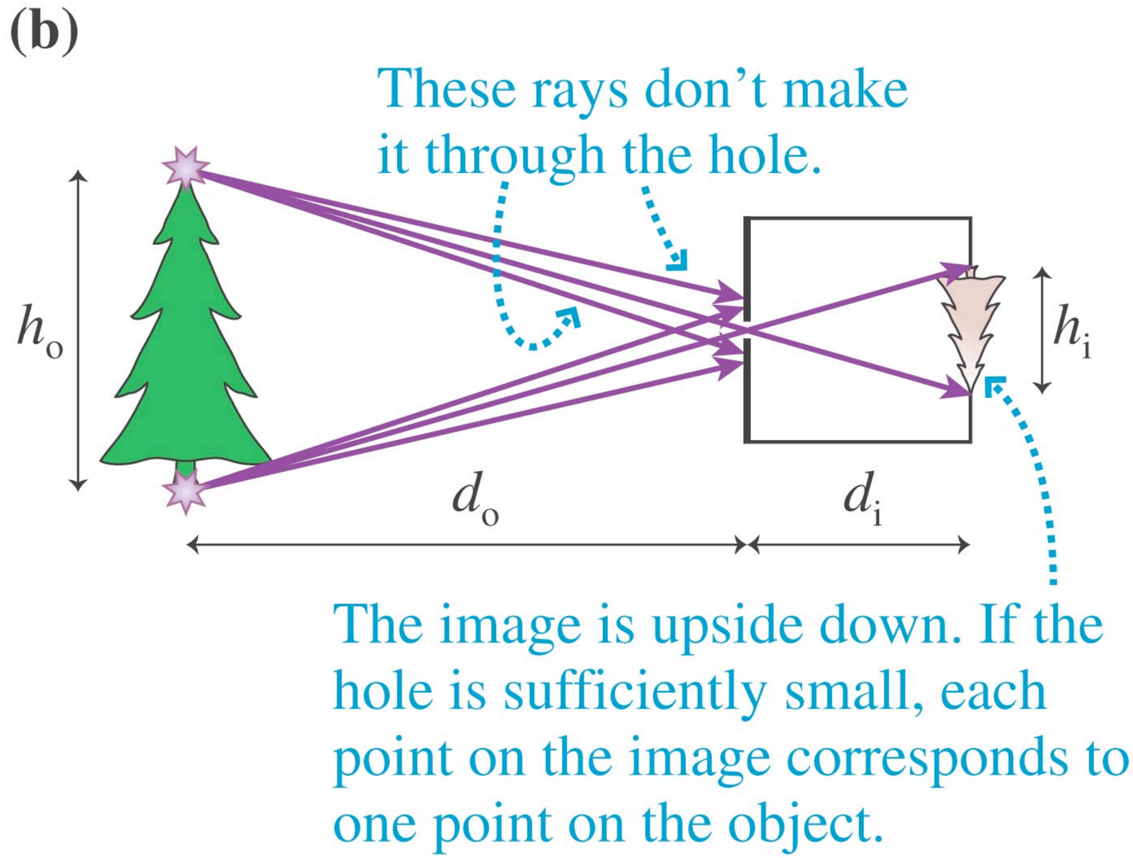

The sun is a self-luminous object.
Emitted light
Reflected light
The tree is a reflective object.
The camera "sees" light rays reflected by the tree but not the rays from the sun.

Source sends out rays in all directions

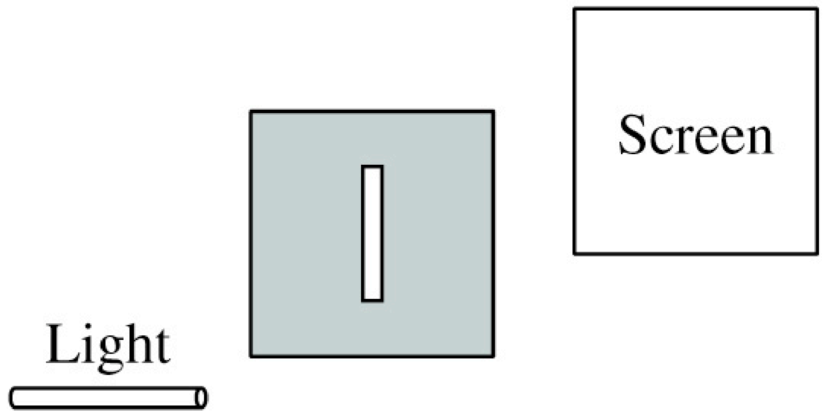


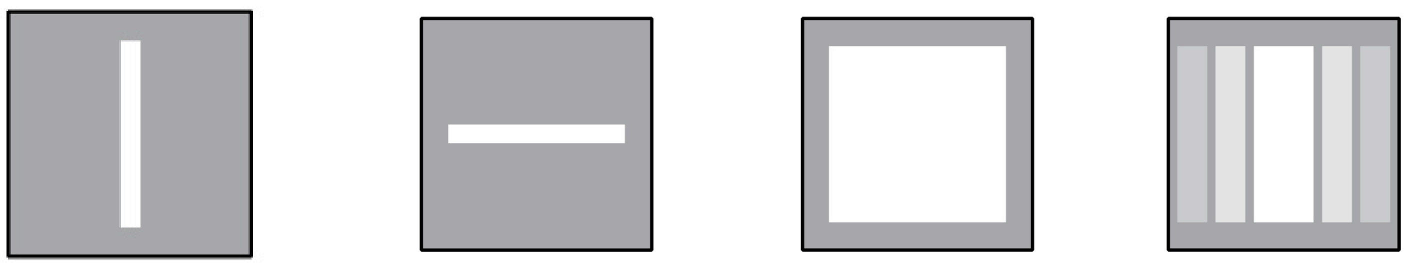
Pin hole camera
(No lens)

$$\frac{h_i}{d_i} = \frac{h_0}{d_0}$$



A long, thin light bulb illuminates a vertical aperture. Which pattern of light do you see on a viewing screen behind the aperture?

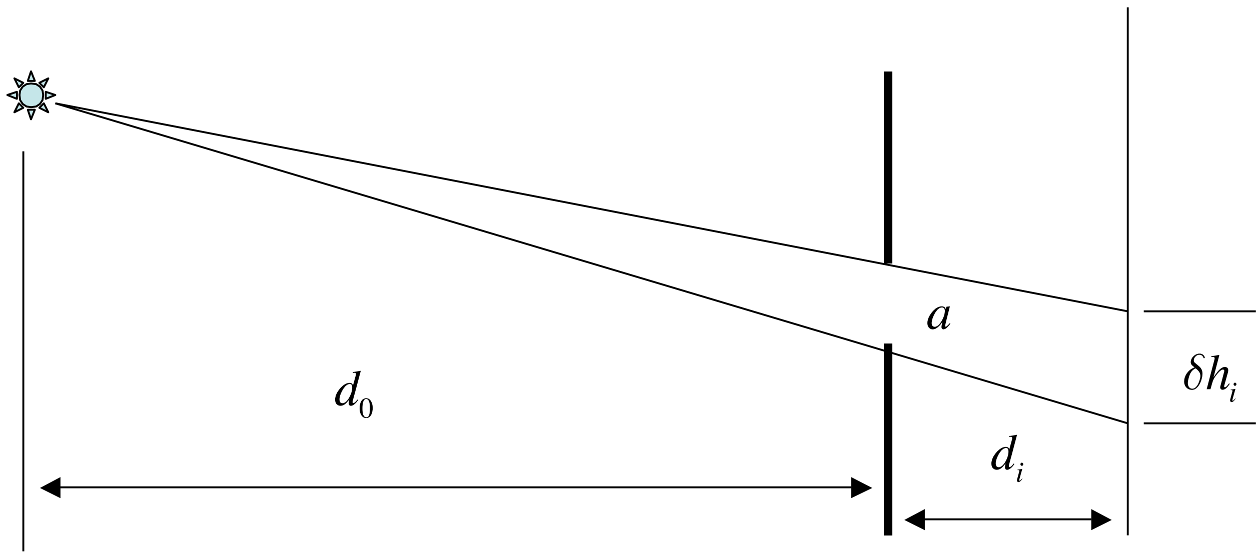




(a) (b) (c) (d)

Pin hole camera
(No lens)

How big and how small can the pin hole be?



Spread in rays from same point on object should be smaller than image

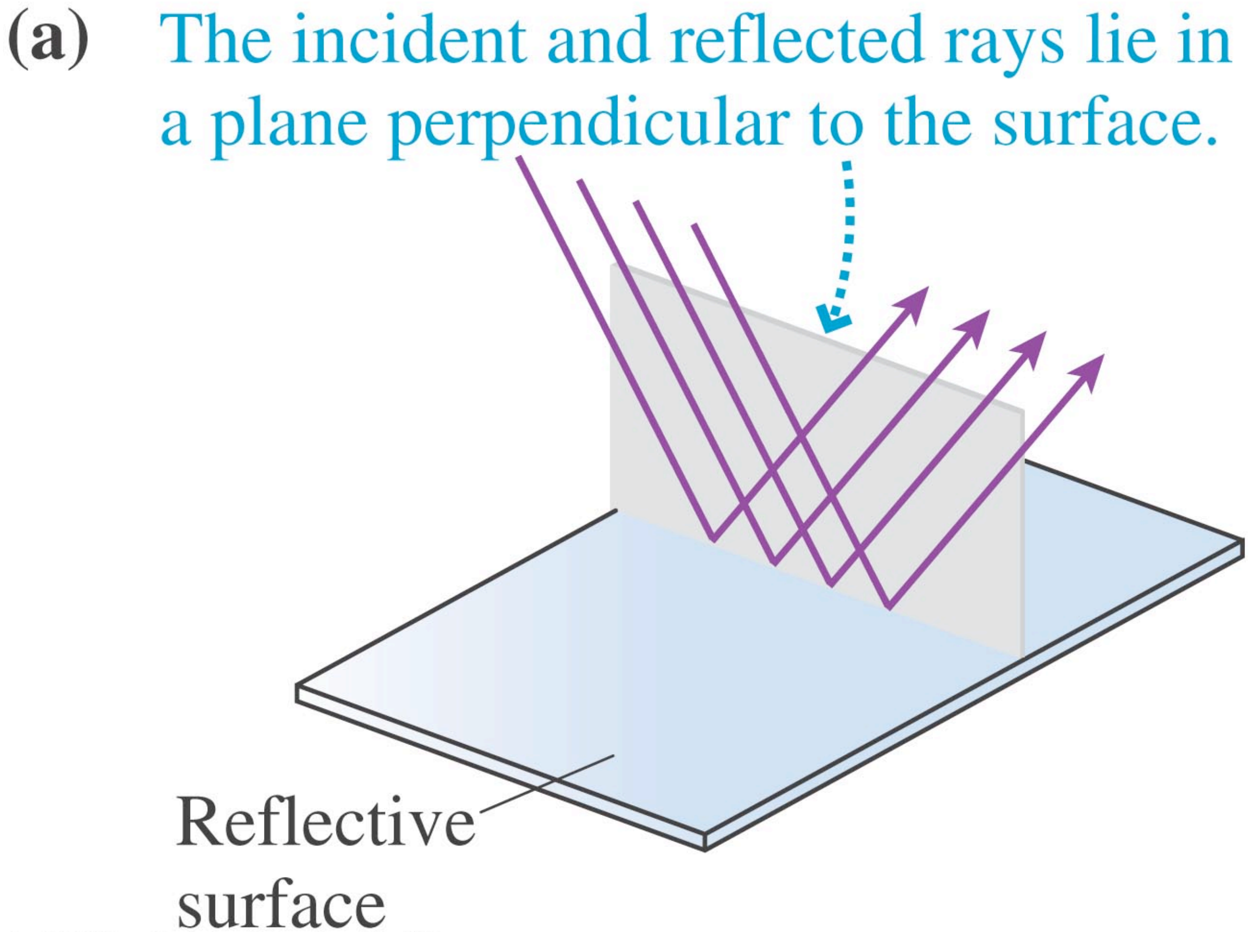
$$\delta h_i = a\frac{d_0 + d_i}{d_0} \simeq a << h_i$$

Diffraction should be negligible

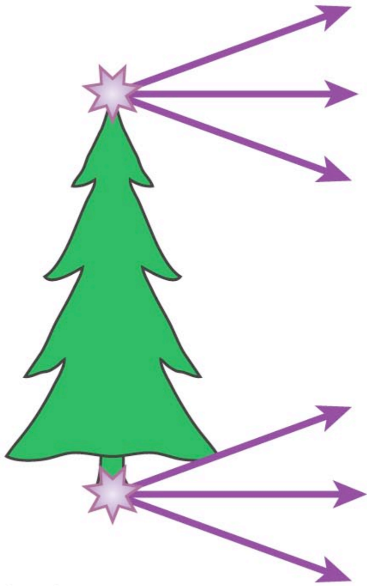
$$a >> \sqrt{\lambda d_i}$$

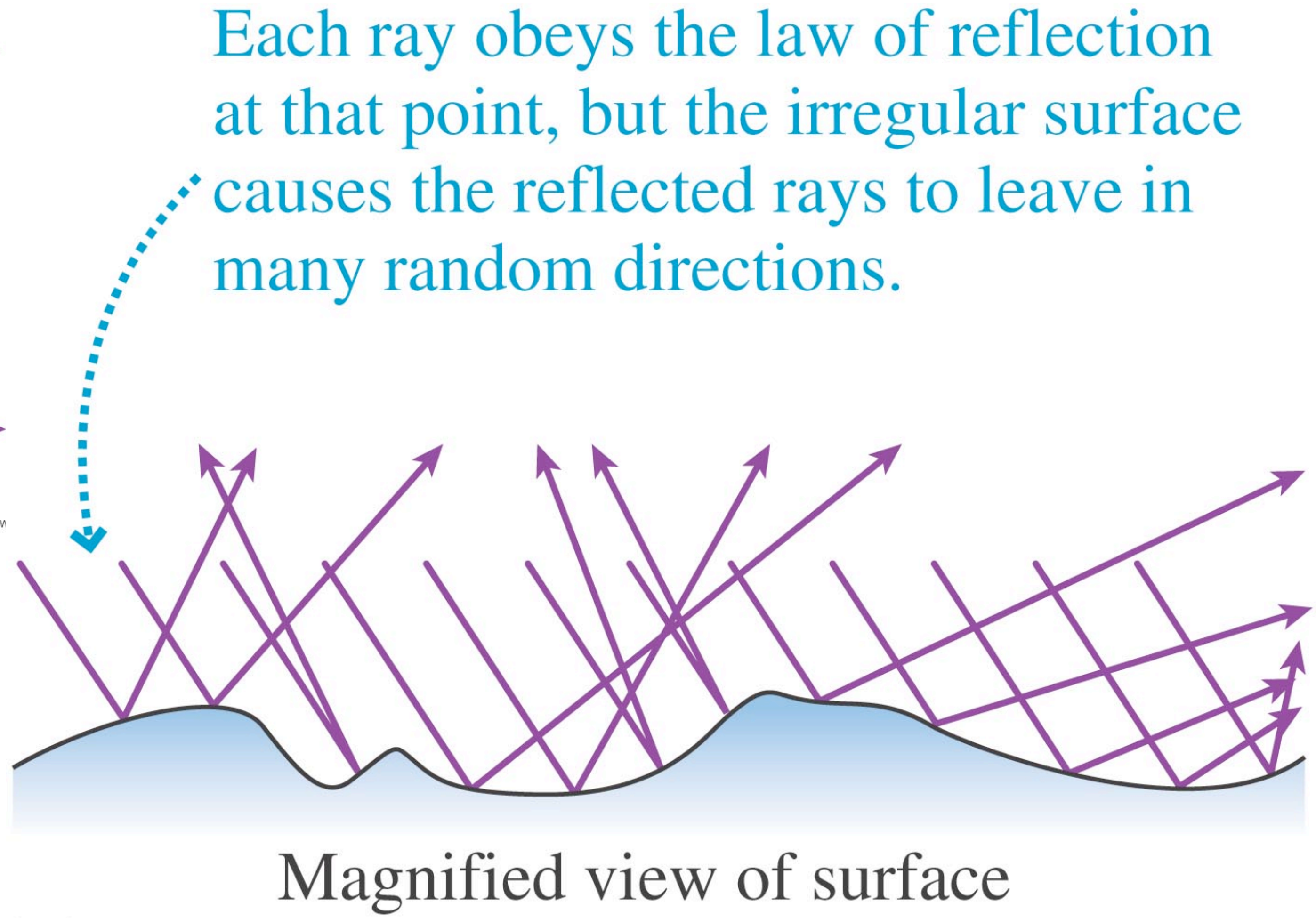
Also, a should be big enough to allow enough light to see.

Specular Reflection - reflection from a smooth surface

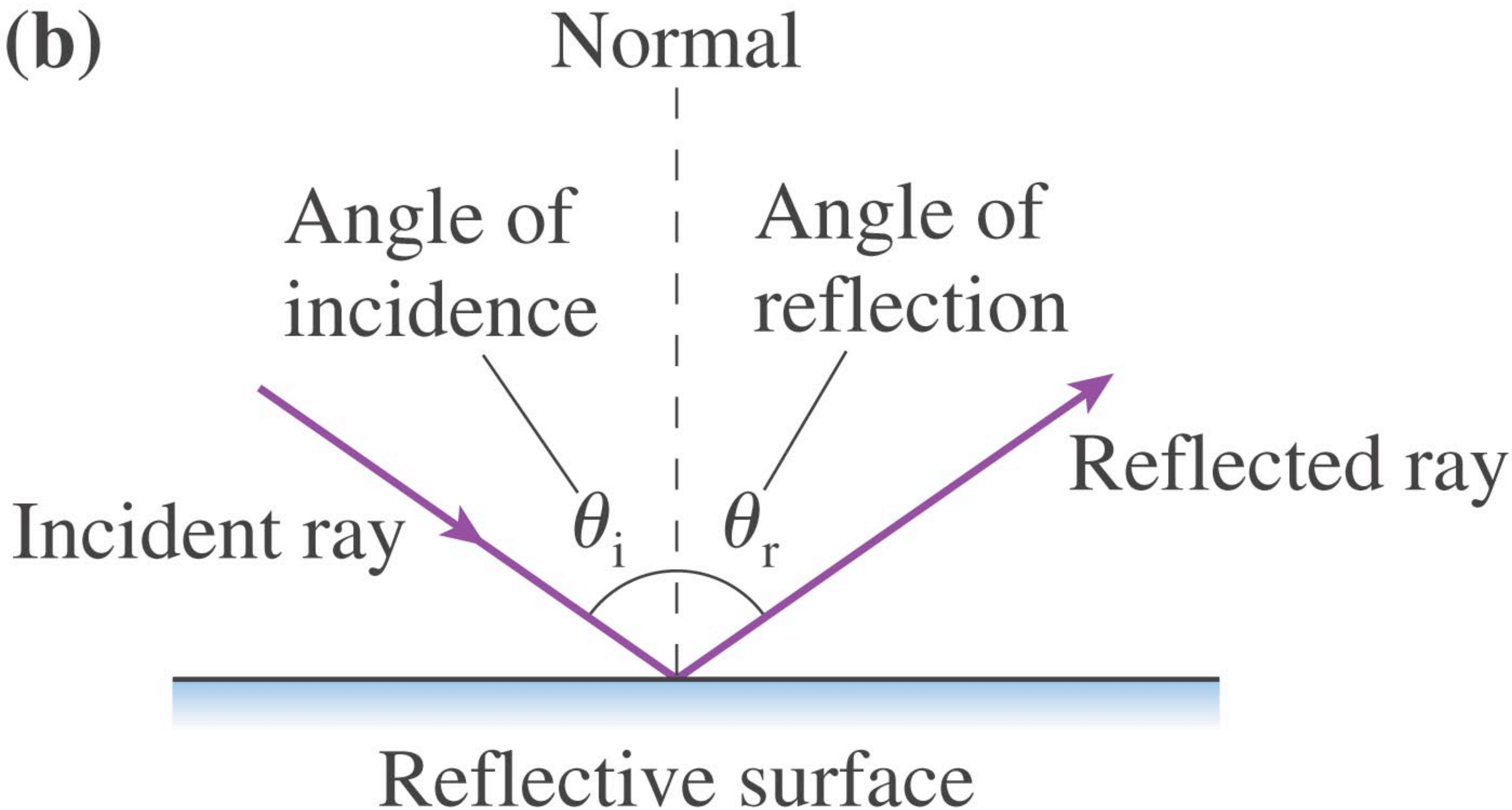


Diffuse Reflection - reflection from an irregular surface

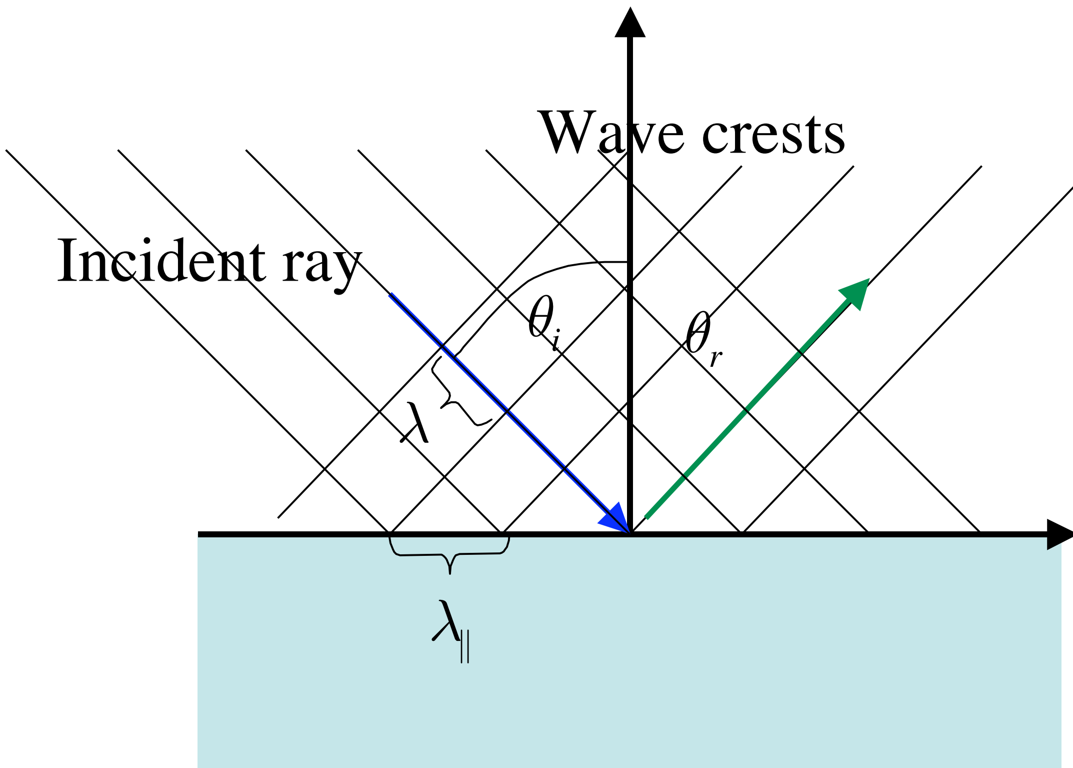




Angle of Incidence = Angle of Reflection



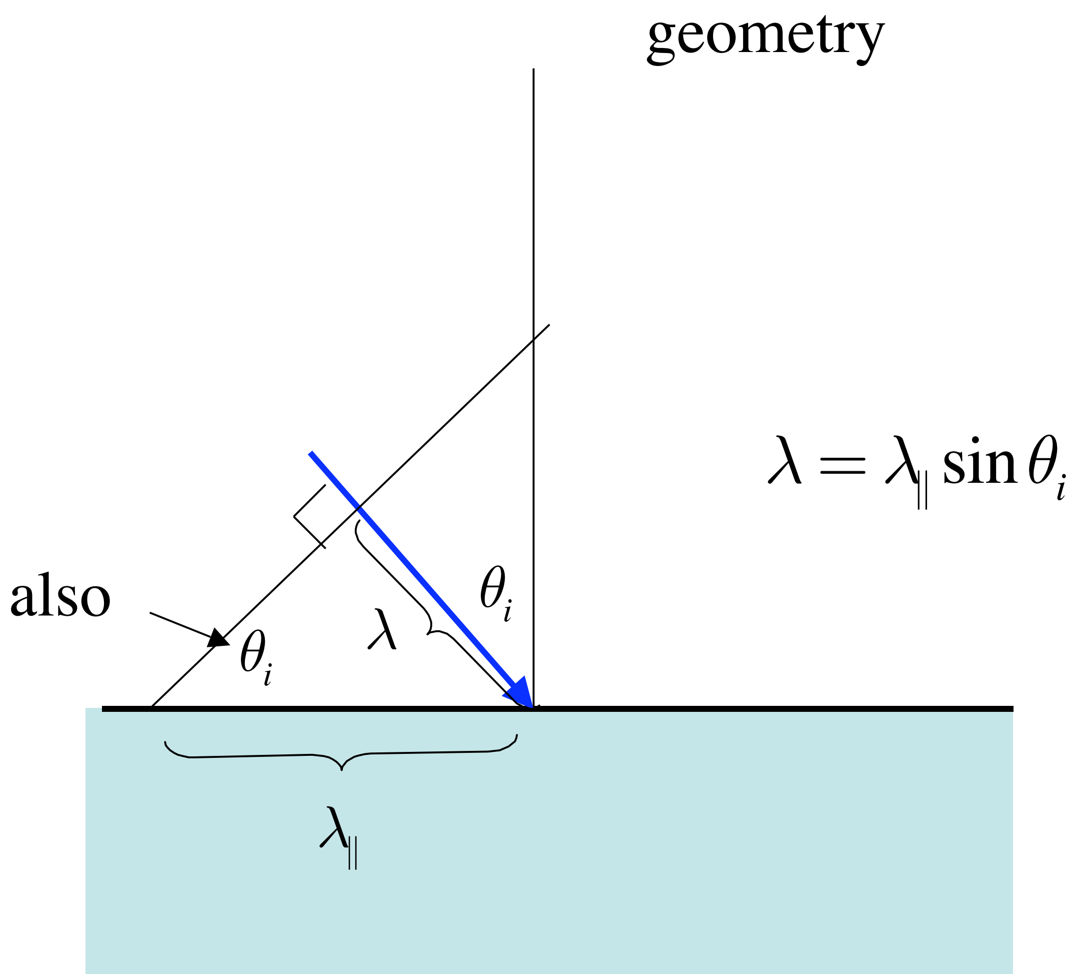
Why does angle of incidence = angle of reflection?

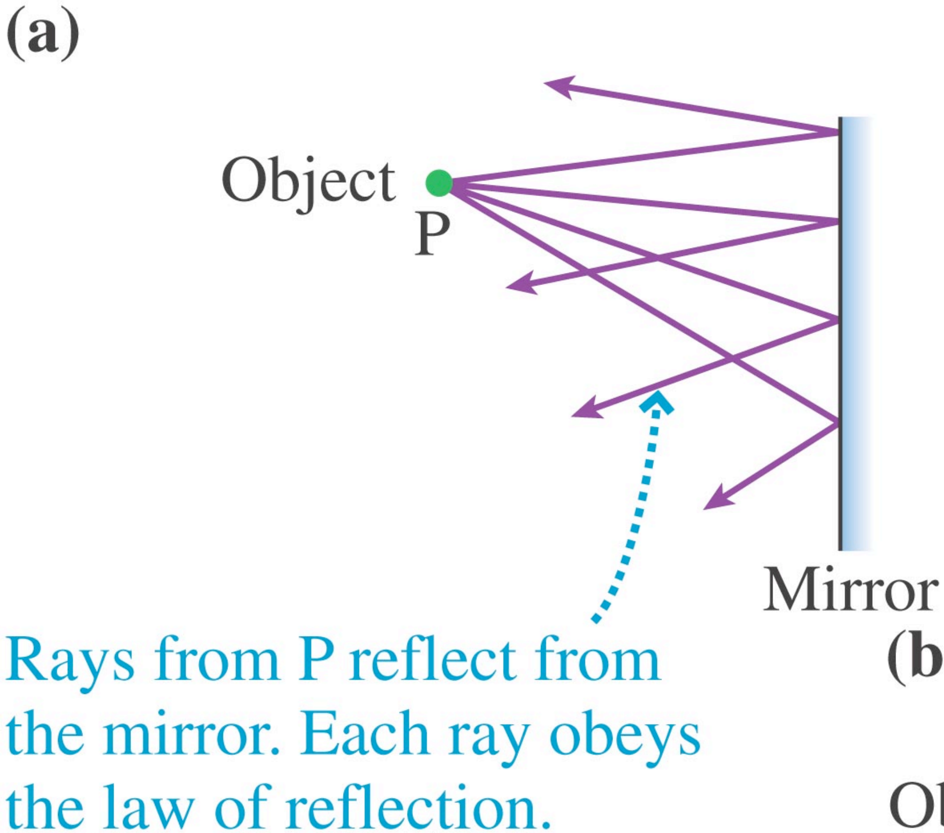


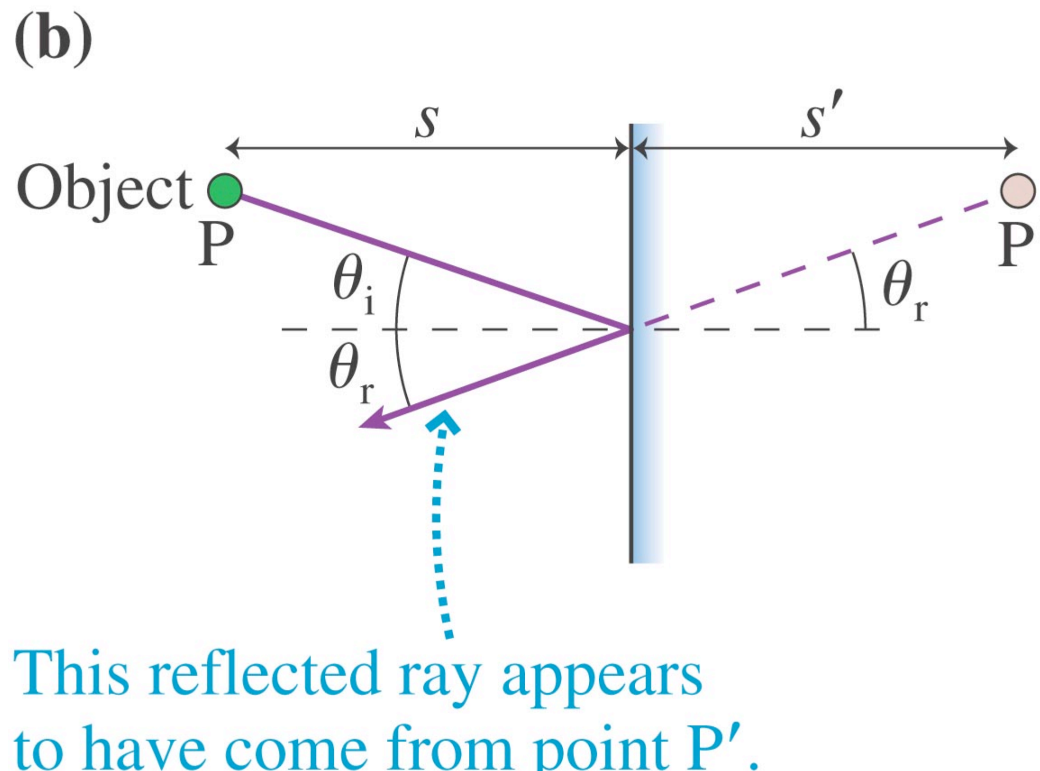
$$\lambda_{\parallel} = \frac{\lambda}{\sin\theta_i} = \frac{\lambda}{\sin\theta_r}$$

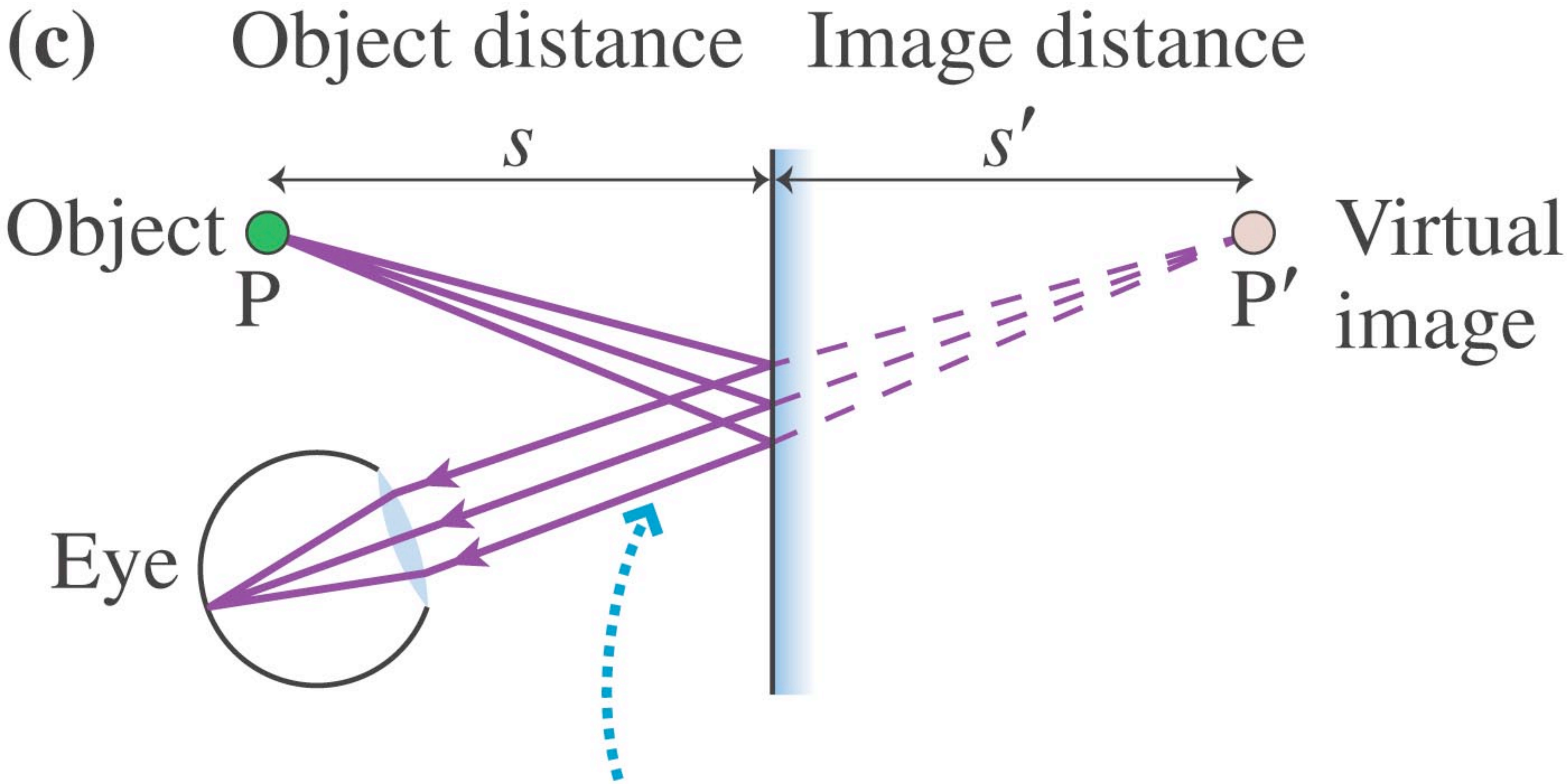
$$\theta_i = \theta_r$$

Incident and Reflected wave crests must match up along surface

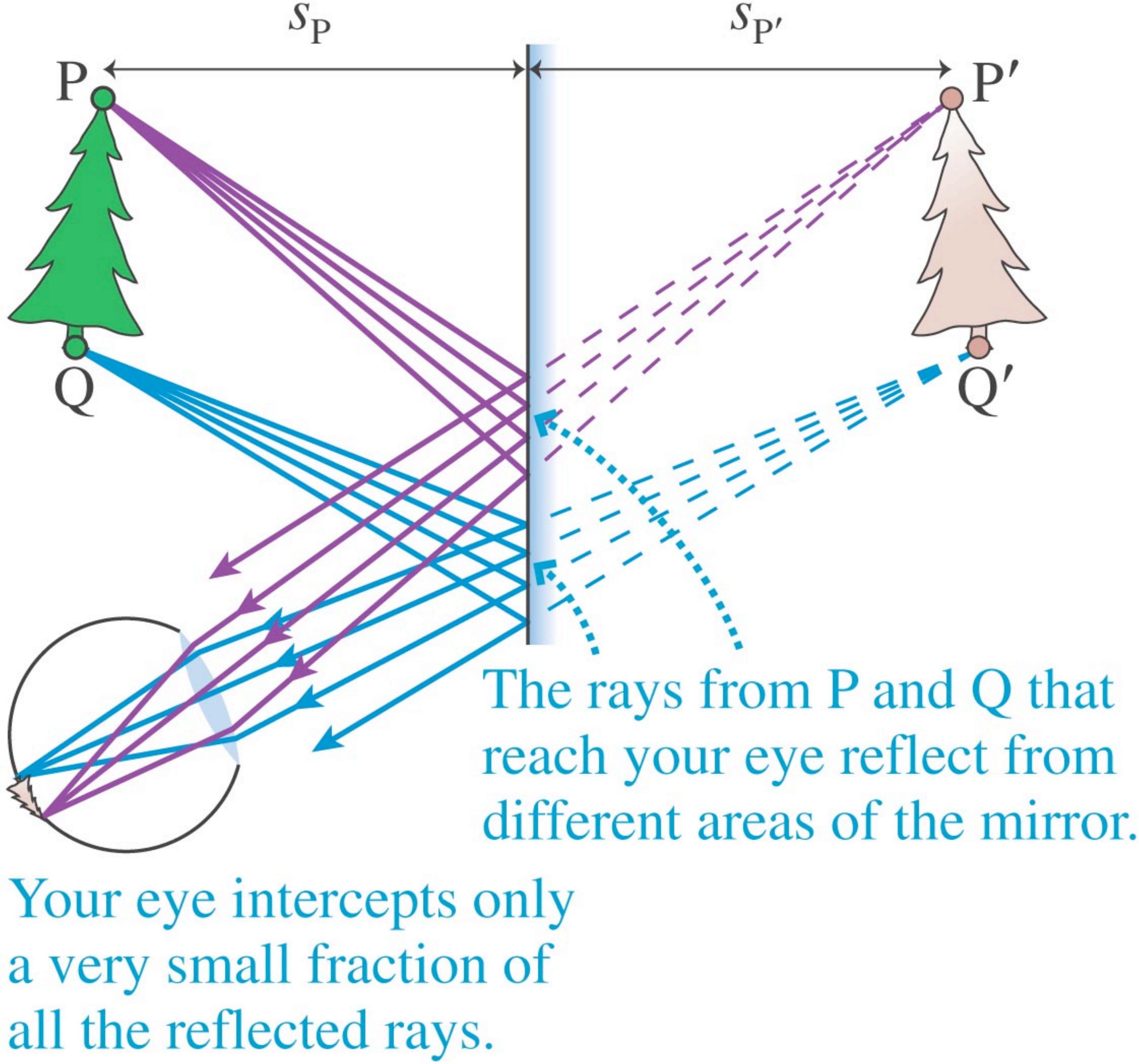

geometry
$\lambda = \lambda_{\|} \sin\theta_i$
also
θ_i
λ
θ_i
$\lambda_{\|}$


(a)
Object
P
Mirror
Rays from P reflect from the mirror. Each ray obeys the law of reflection.


(b)
s
s′
Object
P
P′
θi
θr
θr
This reflected ray appears to have come from point P′.



The reflected rays *all* diverge from P′, which appears to be the source of the reflected rays. Your eye collects the bundle of diverging rays and "sees" the light coming from P′.


s_P
$s_{P'}$
P
P′
Q
Q′
The rays from P and Q that reach your eye reflect from different areas of the mirror.
Your eye intercepts only a very small fraction of all the reflected rays.

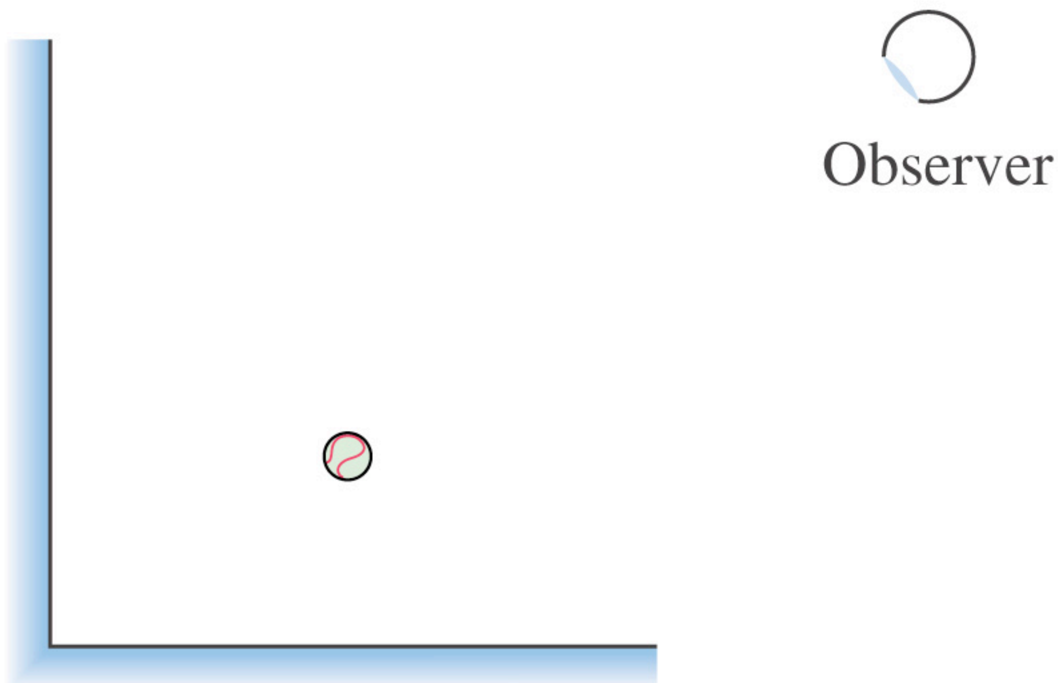
Two plane mirrors form a right angle. How many images of the ball can you see in the mirrors?

A. 1

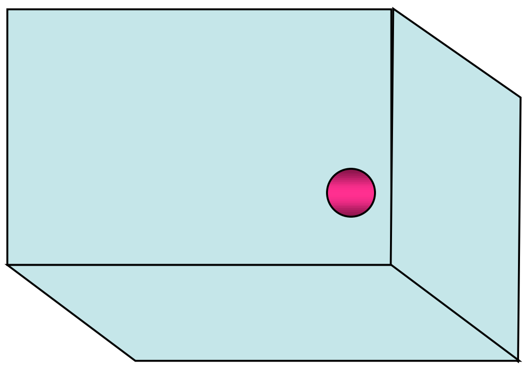
B. 2

C. 3

D. 4



Suppose the corner had a third side.

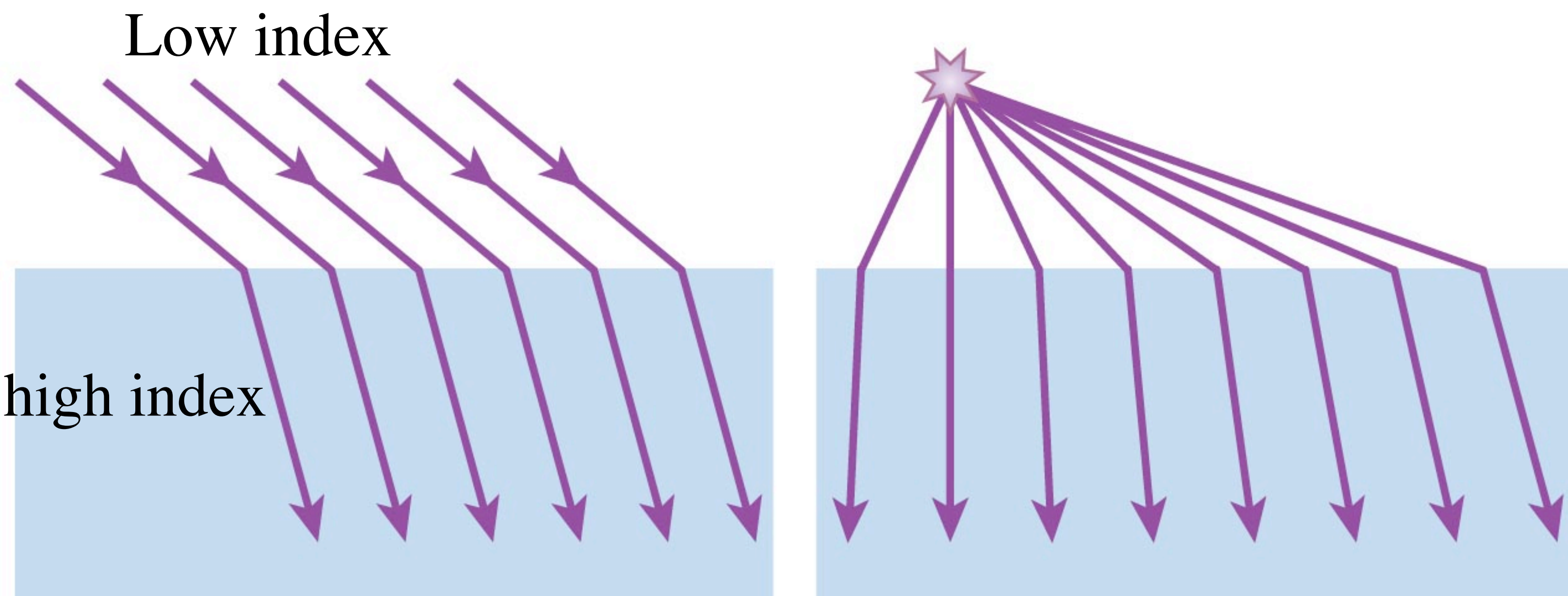


How many images?

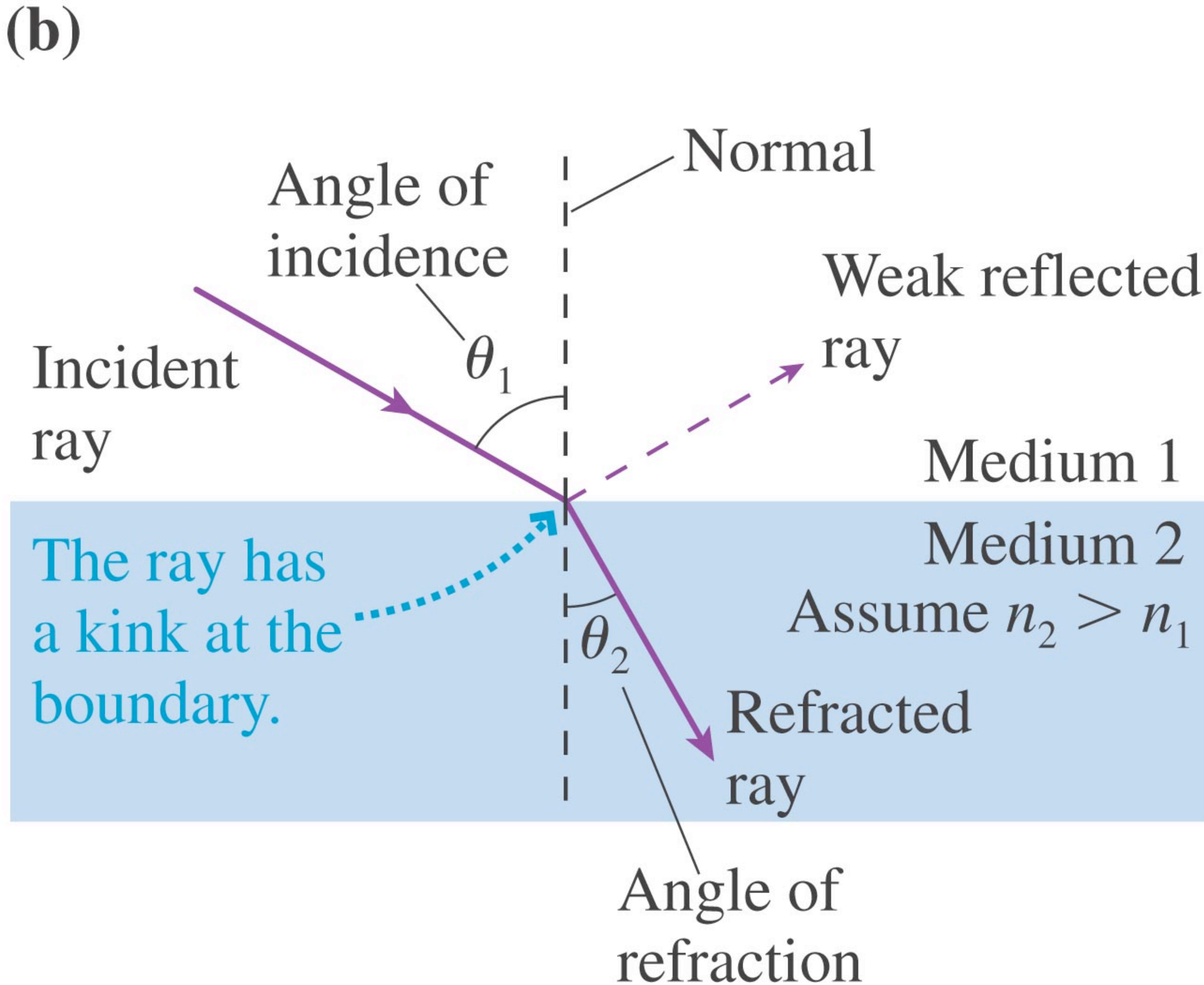
A. 3
B. 6
C. 7
D. 8

(a)

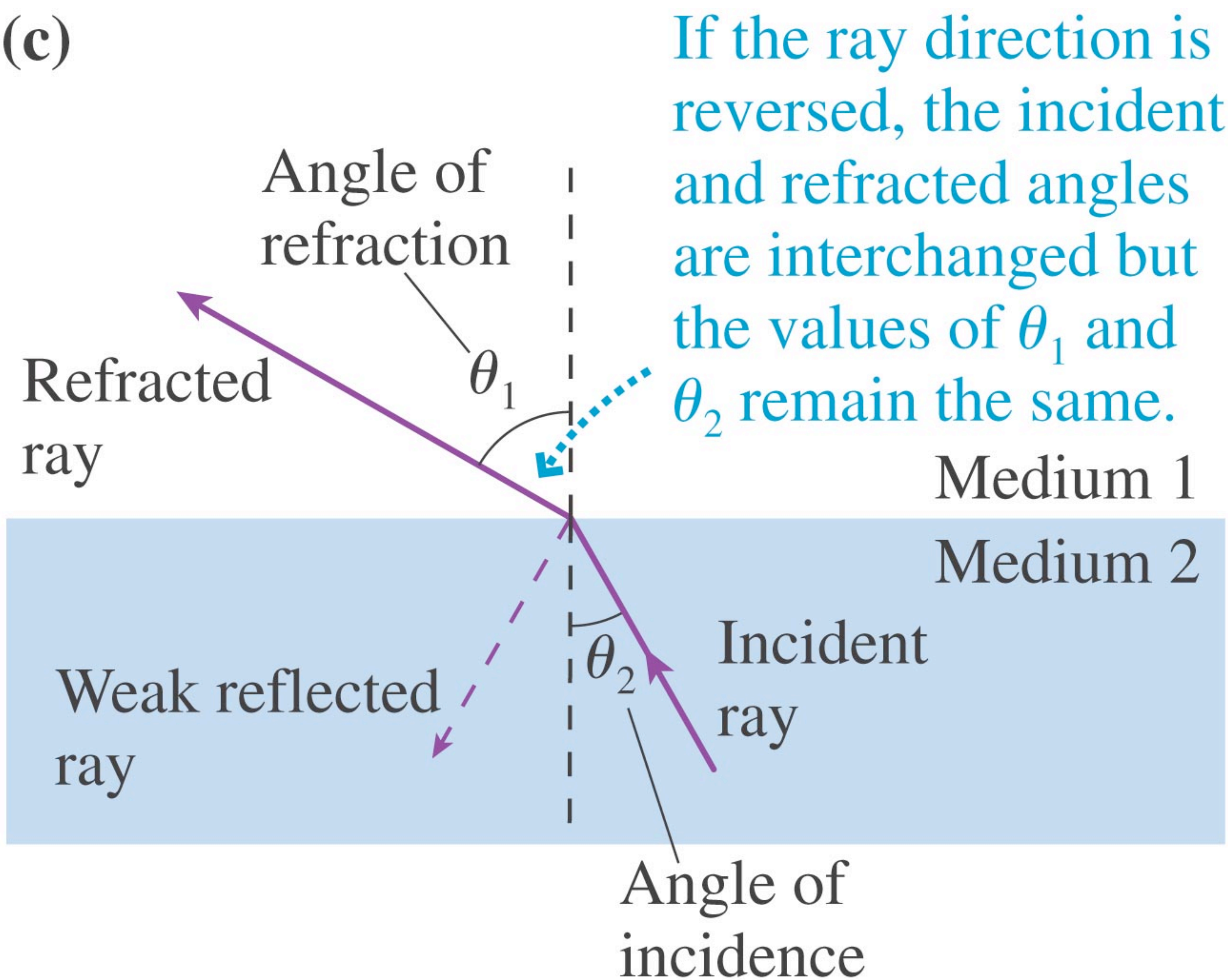
Refraction - path of light bends when going from one medium to another   Depends on index of refraction



Refraction of a parallel beam of light and of rays from a point source


(b)
Normal
Angle of incidence
θ_1
Weak reflected ray
Incident ray
Medium 1
Medium 2
Assume $n_2 > n_1$
The ray has a kink at the boundary.
θ_2
Refracted ray
Angle of refraction

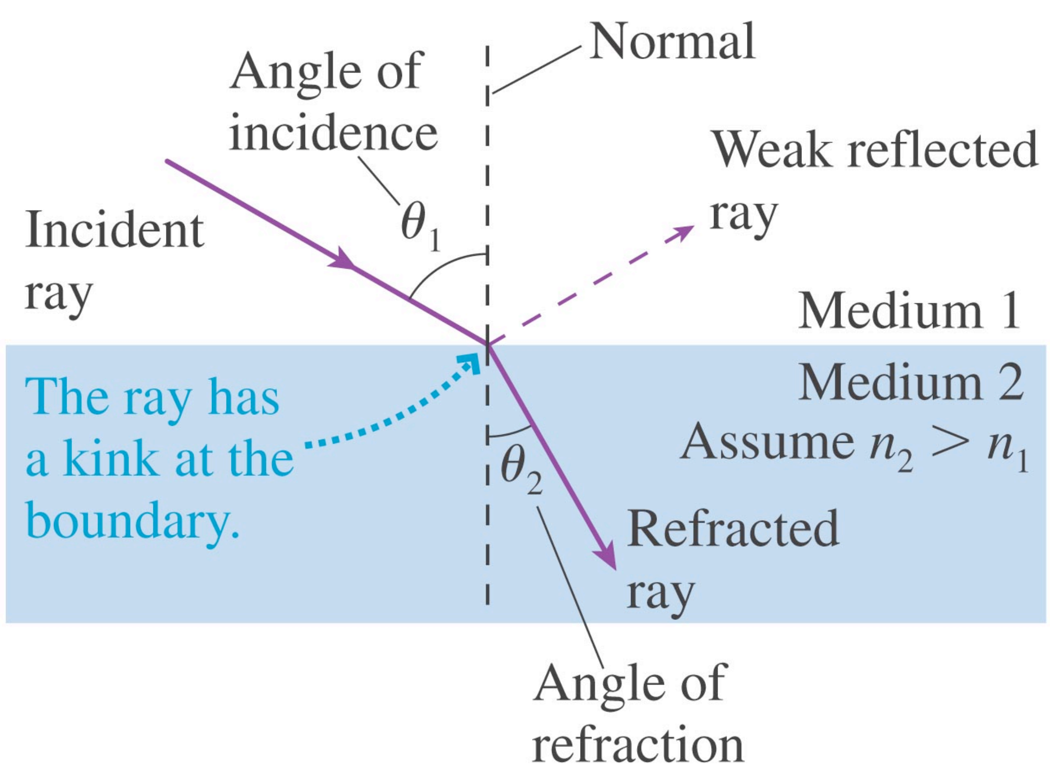
(c)


If the ray direction is reversed, the incident and refracted angles are interchanged but the values of θ_1 and θ_2 remain the same.
Angle of refraction
θ_1
Refracted ray
Medium 1
Medium 2
Weak reflected ray
θ_2
Incident ray
Angle of incidence

Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

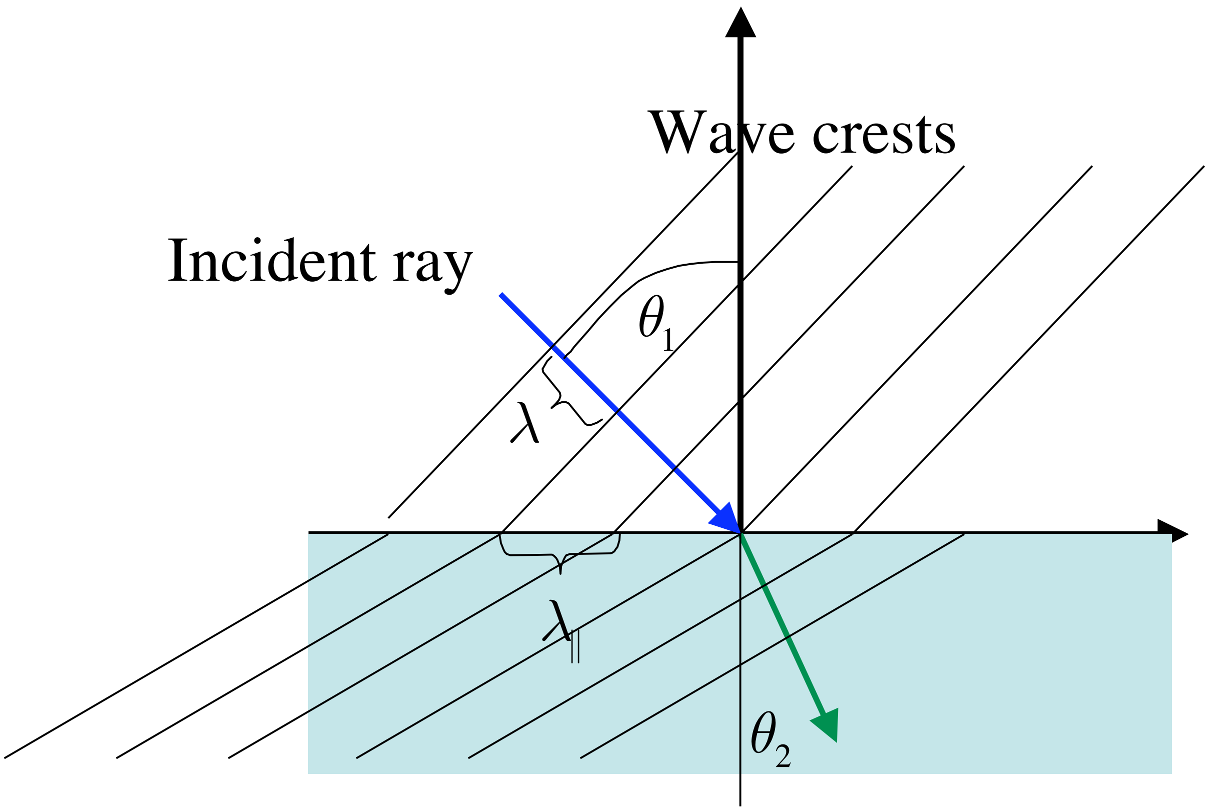
(b)



Remember definition of index of refraction

$$n = \frac{c}{v_{em}}$$

Why Snell's Law?



$$\lambda_\parallel = \frac{\lambda_1}{\sin\theta_1} = \frac{\lambda_2}{\sin\theta_2}$$

$$\lambda_1 = \frac{\lambda_{vac}}{n_1}$$

$$\lambda_2 = \frac{\lambda_{vac}}{n_2}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Incident and Transmitted wave crests must match up along surface

TABLE 23.1 Indices of refraction

Medium	n
Vacuum	1.00 exactly
Air (actual)	1.0003
Air (accepted)	1.00
Water	1.33
Ethyl alcohol	1.36
Oil	1.46
Glass (typical)	1.50
Polystyrene plastic	1.59
Cubic zirconia	2.18
Diamond	2.41
Silicon (infrared)	3.50

For most material
$n>1$

Plasma
$n<1$

On Mastering Physics Homework you are to pretend that plasma does not exist

Tactics: Analyzing refraction

TACTICS BOX 23.1 **Analyzing refraction**

1. **Draw a ray diagram.** Represent the light beam with one ray.
2. **Draw a line normal to the boundary.** Do this at each point where the ray intersects a boundary.
3. **Show the ray bending in the correct direction.** The angle is larger on the side with the smaller index of refraction. This is the qualitative application of Snell's law.
4. **Label angles of incidence and refraction.** Measure all angles from the normal.
5. **Use Snell's law.** Calculate the unknown angle or unknown index of refraction.

Exercises 11–15

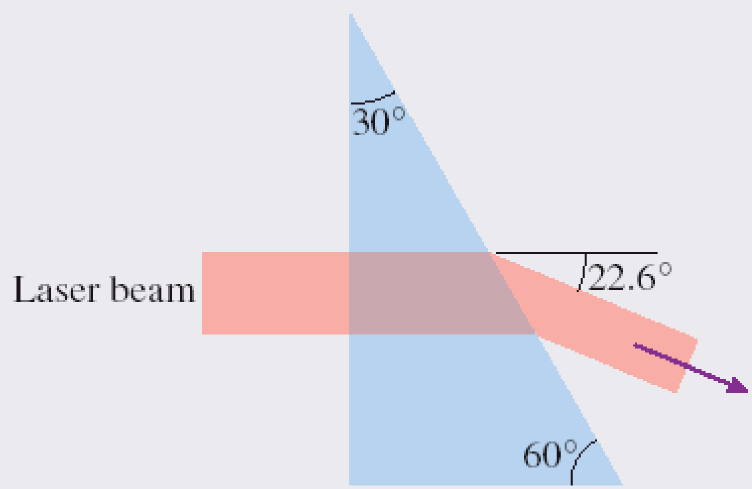
EXAMPLE 23.4 Measuring the index of refraction

QUESTION:

EXAMPLE 23.4 Measuring the index of refraction

FIGURE 23.19 shows a laser beam deflected by a 30°-60°-90° prism. What is the prism's index of refraction?

FIGURE 23.19 A prism deflects a laser beam.



EXAMPLE 23.4 Measuring the index of refraction

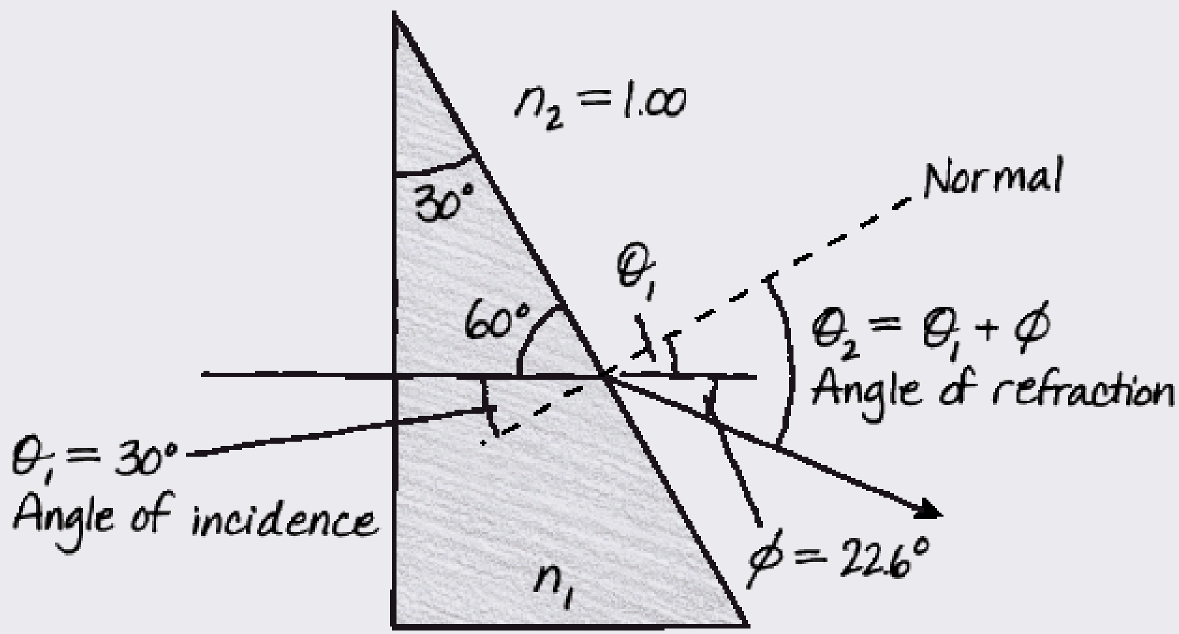
MODEL Represent the laser beam with a single ray and use the ray model of light.

EXAMPLE 23.4 Measuring the index of refraction

VISUALIZE **FIGURE 23.20** uses the steps of Tactics Box 23.1 to draw a ray diagram. The ray is incident perpendicular to the front face of the prism ($\theta_{\text{incident}} = 0°$), thus it is transmitted through the first boundary without deflection. At the second boundary it is especially important to *draw the normal to the surface* at the point of incidence and to *measure angles from the normal.*

EXAMPLE 23.4 Measuring the index of refraction

FIGURE 23.20 Pictorial representation of a laser beam passing through the prism.



θ_1 and θ_2 are measured from the normal.

EXAMPLE 23.4 Measuring the index of refraction

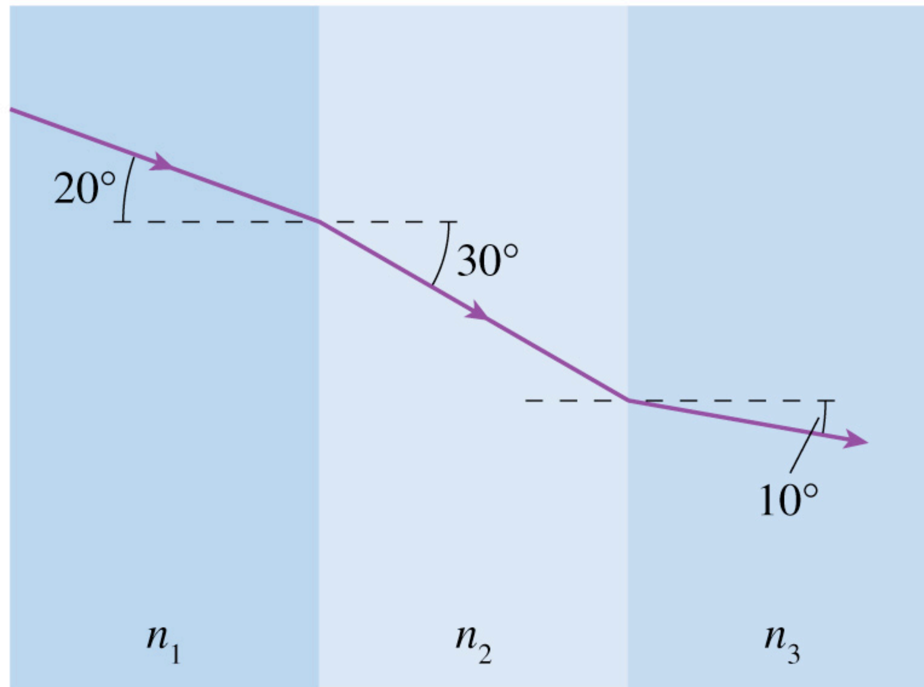
SOLVE From the geometry of the triangle you can find that the laser's angle of incidence on the hypotenuse of the prism is $\theta_1 = 30°$, the same as the apex angle of the prism. The ray exits the prism at angle θ_2 such that the deflection is $\phi = \theta_2 - \theta_1 = 22.6°$. Thus $\theta_2 = 52.6°$. Knowing both angles and $n_2 = 1.00$ for air, we can use Snell's law to find n_1:

$$n_1 = \frac{n_2 \sin\theta_2}{\sin\theta_1} = \frac{1.00 \sin 52.6°}{\sin 30°} = 1.59$$

EXAMPLE 23.4 Measuring the index of refraction

ASSESS Referring to the indices of refraction in Table 23.1, we see that the prism is made of plastic.

A light ray travels from medium 1 to medium 3 as shown.
For these media,



A. $n_3 = n_1$.

B. $n_3 > n_1$.

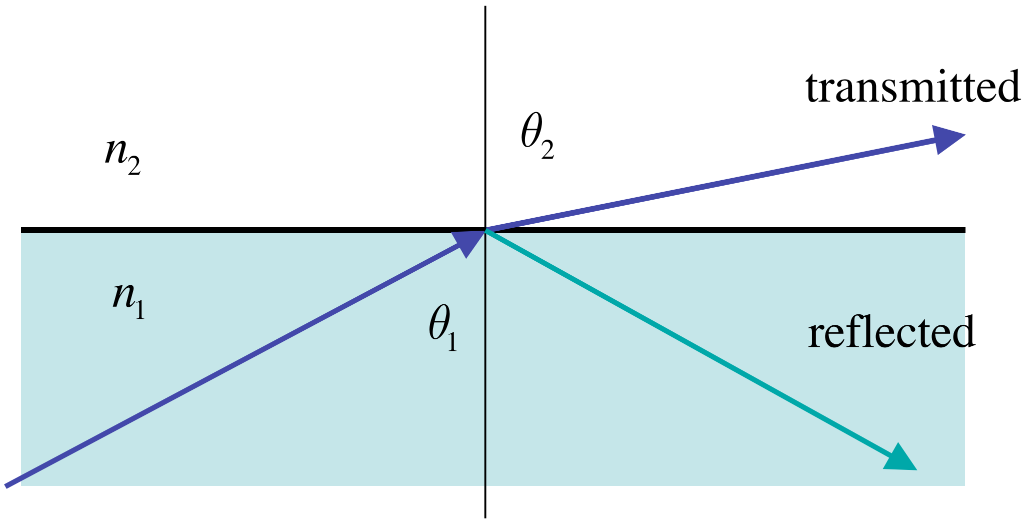
C. $n_3 < n_1$.

D. We can't compare n_1 to n_3 without knowing n_2.

Total Internal Reflection

Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



Based on picture, which is bigger?

A. n_1
B. n_2

Solve for θ_2

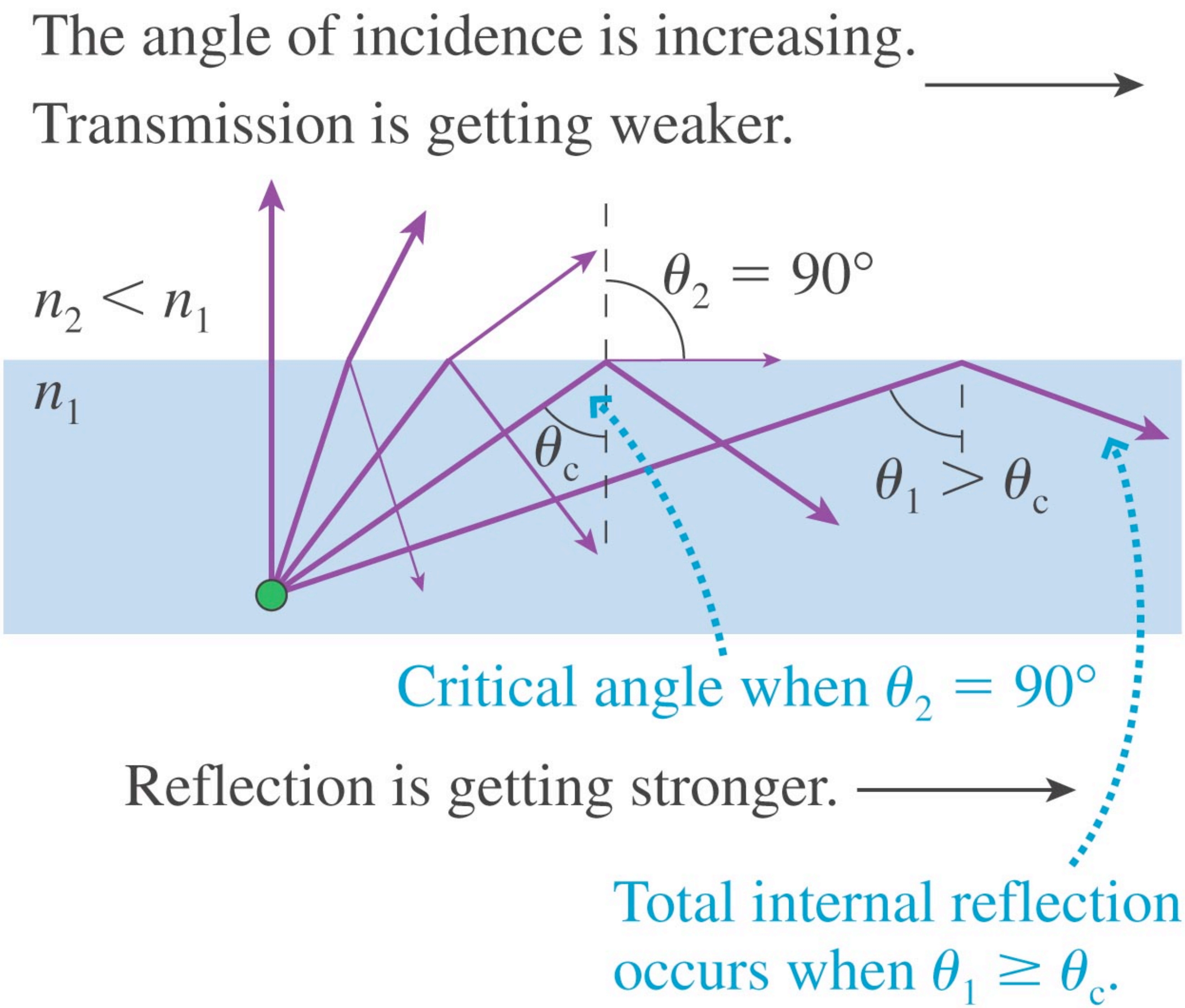
$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

What if $\frac{n_1}{n_2}\sin\theta_1 > 1$?

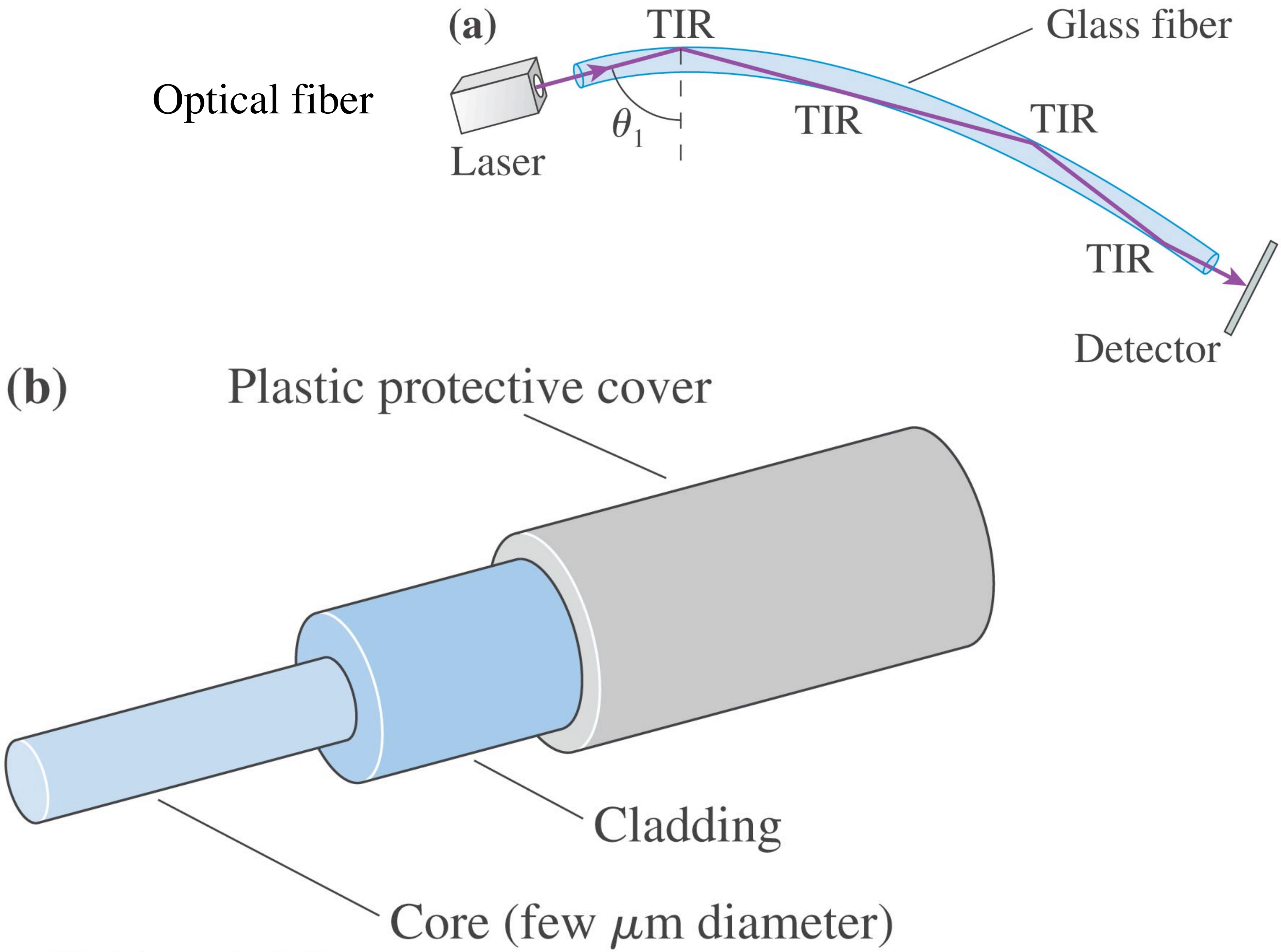
Then there is no θ_2 satisfying SL - no transmission - total reflection

Can only happen if wave is incident from high index material, viz. $n_1 > n_2$.

Critical angle $$\frac{n_1}{n_2}\sin\theta_c = 1$$

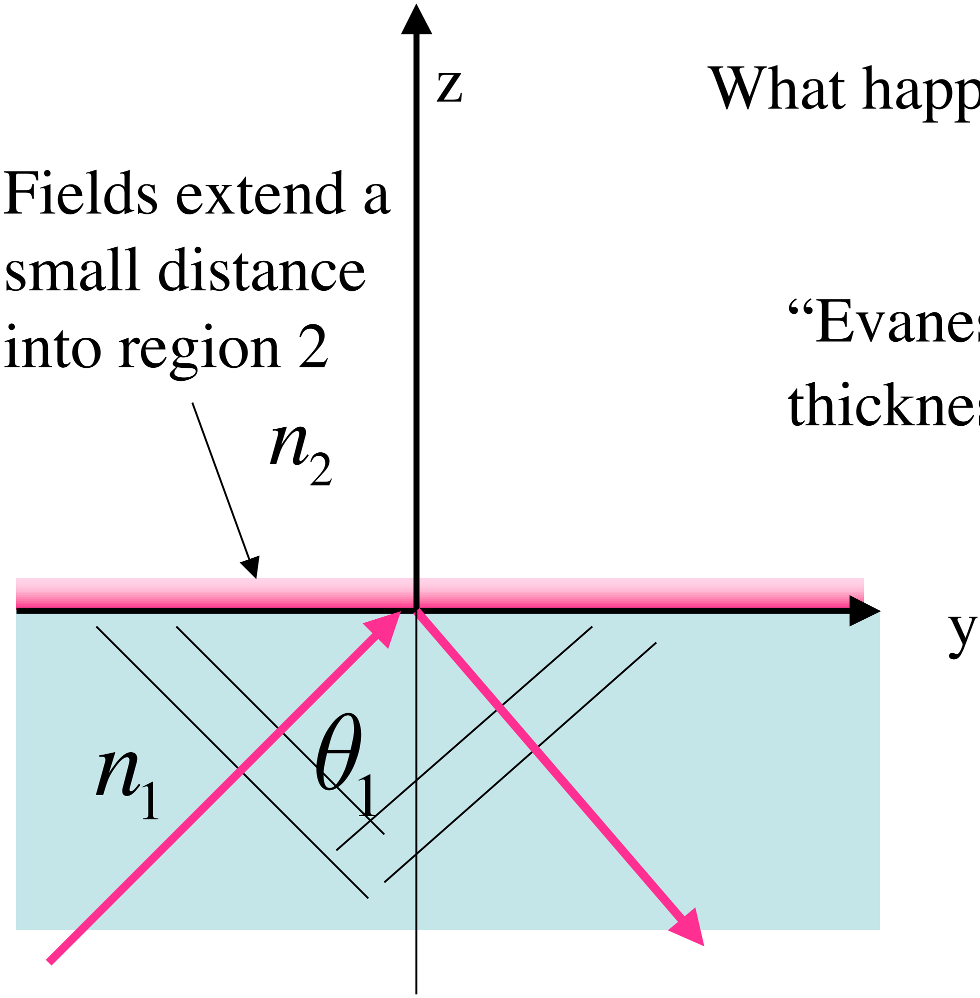


Optical fiber



What happens when $\theta_1 > \theta_c$?

Fields extend a small distance into region 2



"Evanescent" layer thickness δ

$$E \propto \exp[-z/\delta]$$

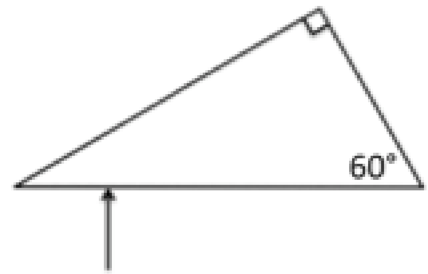
$$\delta = \frac{\lambda_1}{2\pi\sqrt{\sin^2\theta_1 - \sin^2\theta_c}}$$

$$\delta \rightarrow \infty$$

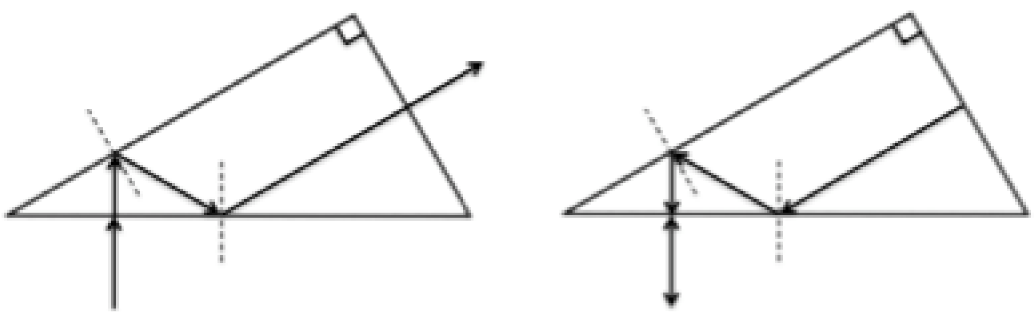
as

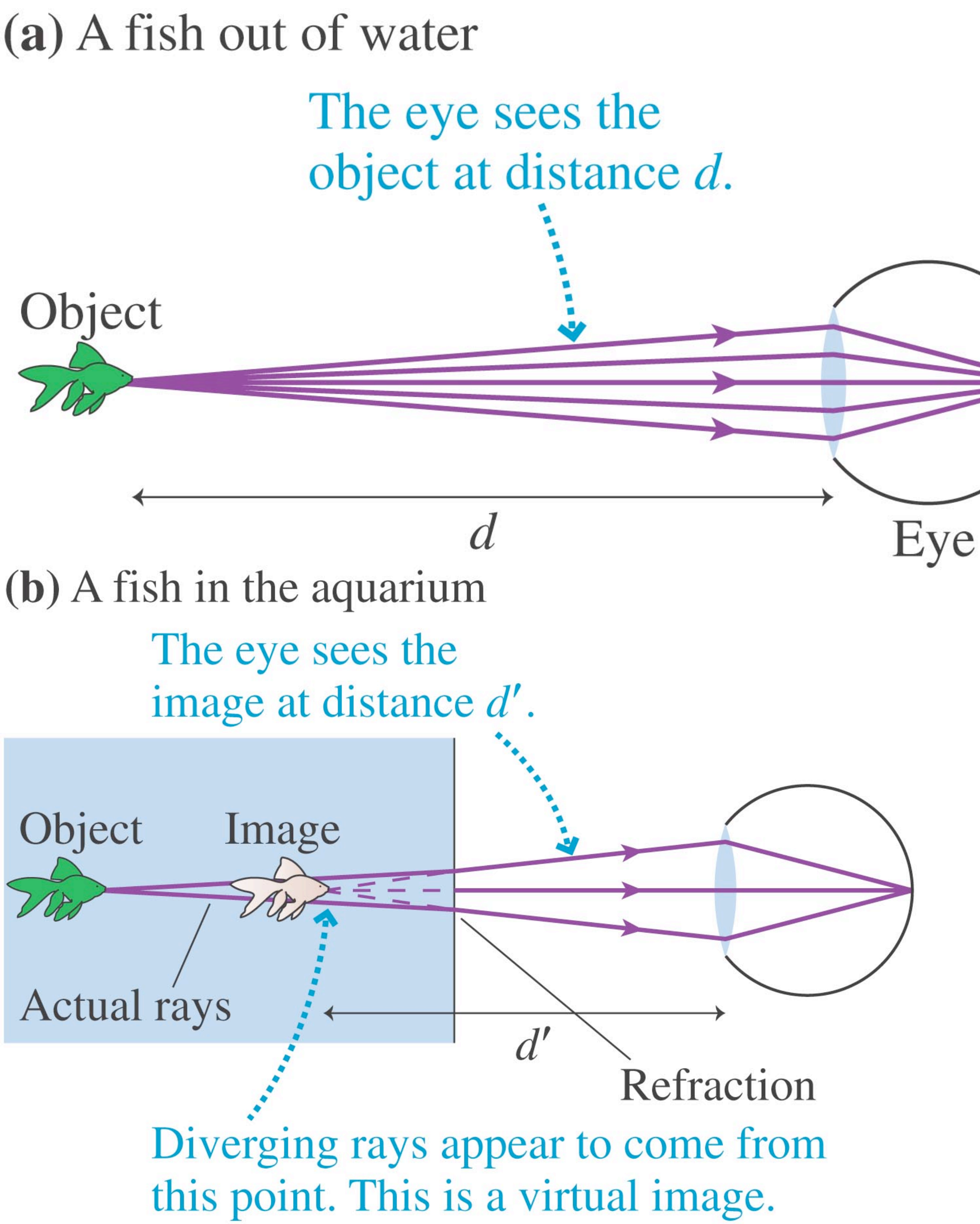
$$\theta_1 \rightarrow \theta_c$$

A light ray traveling in air enters a 30°-60°-90° prism along normal direction to its hypotenuse face, as shown in the figure. The index of refraction of the prism is n =2.1Determine ALL possible outgoing ray directions.

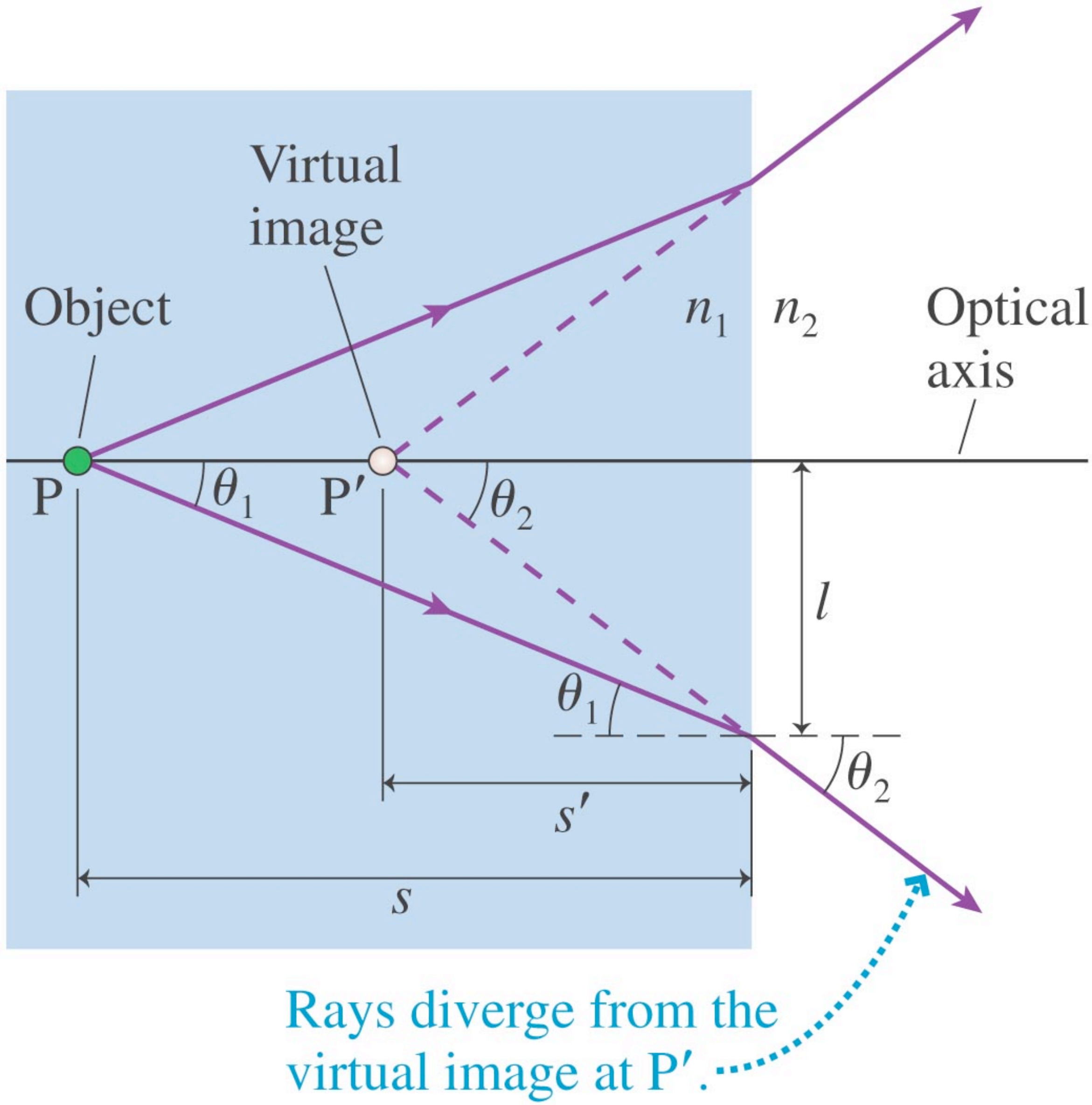


Since n=2.1 and n_{air}≅1, critical angle $\theta_c = \sin^{-1}\left(\frac{n_{air}}{n}\right) = \sin^{-1}\left(\frac{1}{2.1}\right) = 28.42^\circ < 30^\circ$





Virtual image formed due to refraction



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$l = s\tan\theta_1 = s'\tan\theta_2$$

Approximation for small angles:

$$\sin\theta \simeq \tan\theta$$

$$\frac{s'}{s} = \frac{n_2}{n_1}$$

Color

Different colors are associated with light of different wavelengths. The longest wavelengths are perceived as red light and the shortest as violet light. Table 23.2 is a brief summary of the *visible spectrum* of light.

TABLE 23.2 A brief summary of the visible spectrum of light

Color	Approximate wavelength
Deepest red	700 nm
Red	650 nm
Green	550 nm
Blue	450 nm
Deepest violet	400 nm

Different colors are associated with light of different wavelengths.

However, color is a perception, and most of that perception is based on the way our eyes and brain work.

For example combinations of light with different wavelengths appear to have colors different from those of the original components.

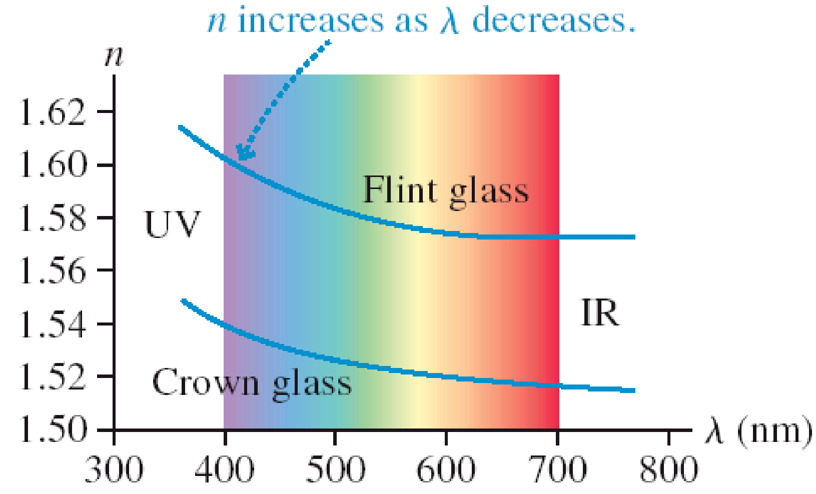
See Chapter 24.3

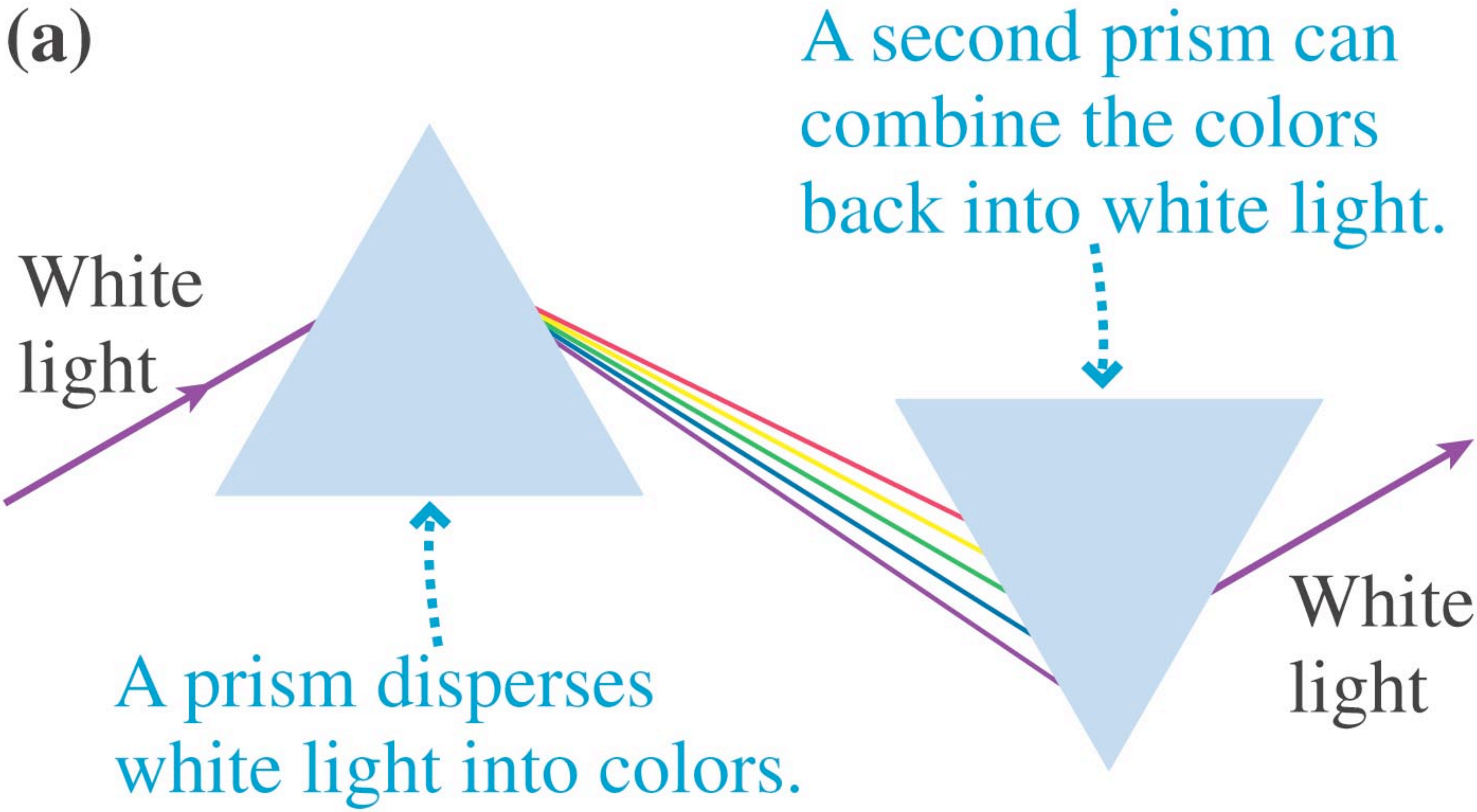
We will focus n the inherent properties of light, not on the way we perceive it.

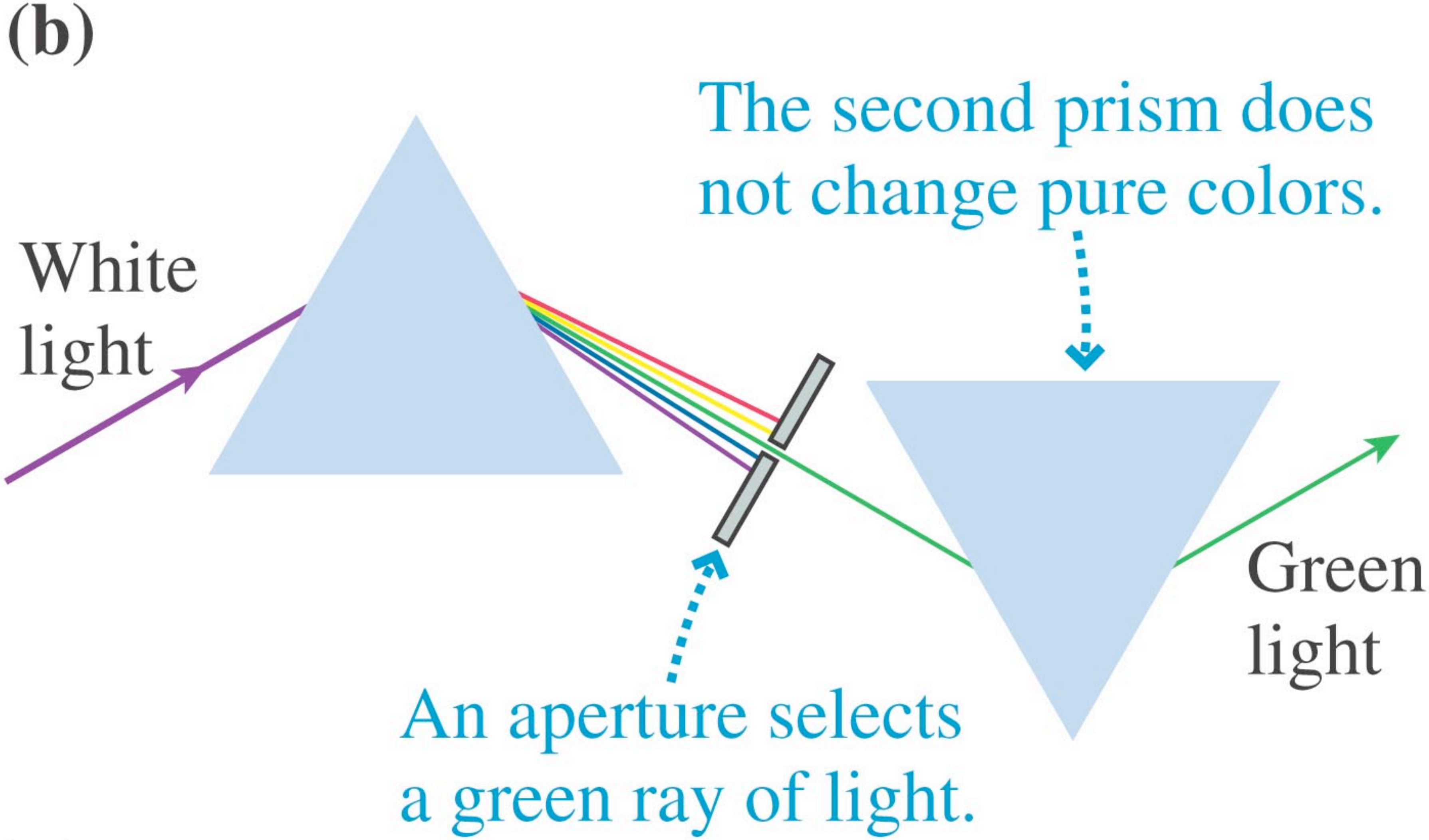
Dispersion

The slight variation of index of refraction with wavelength is known as **dispersion**. Shown is the dispersion curves of two common glasses. Notice that ***n* is larger when the wavelength is shorter**, thus violet light refracts more than red light.

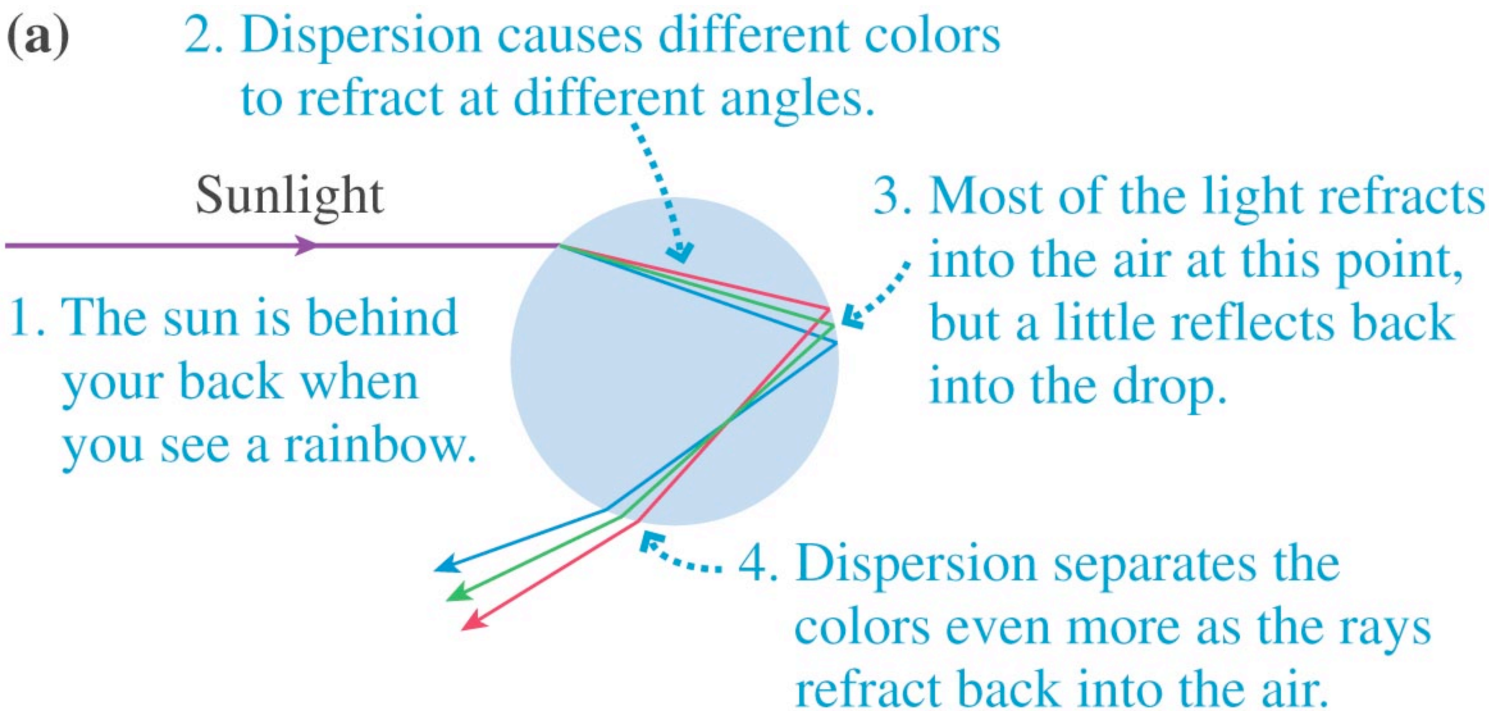
FIGURE 23.29 Dispersion curves show how the index of refraction varies with wavelength.

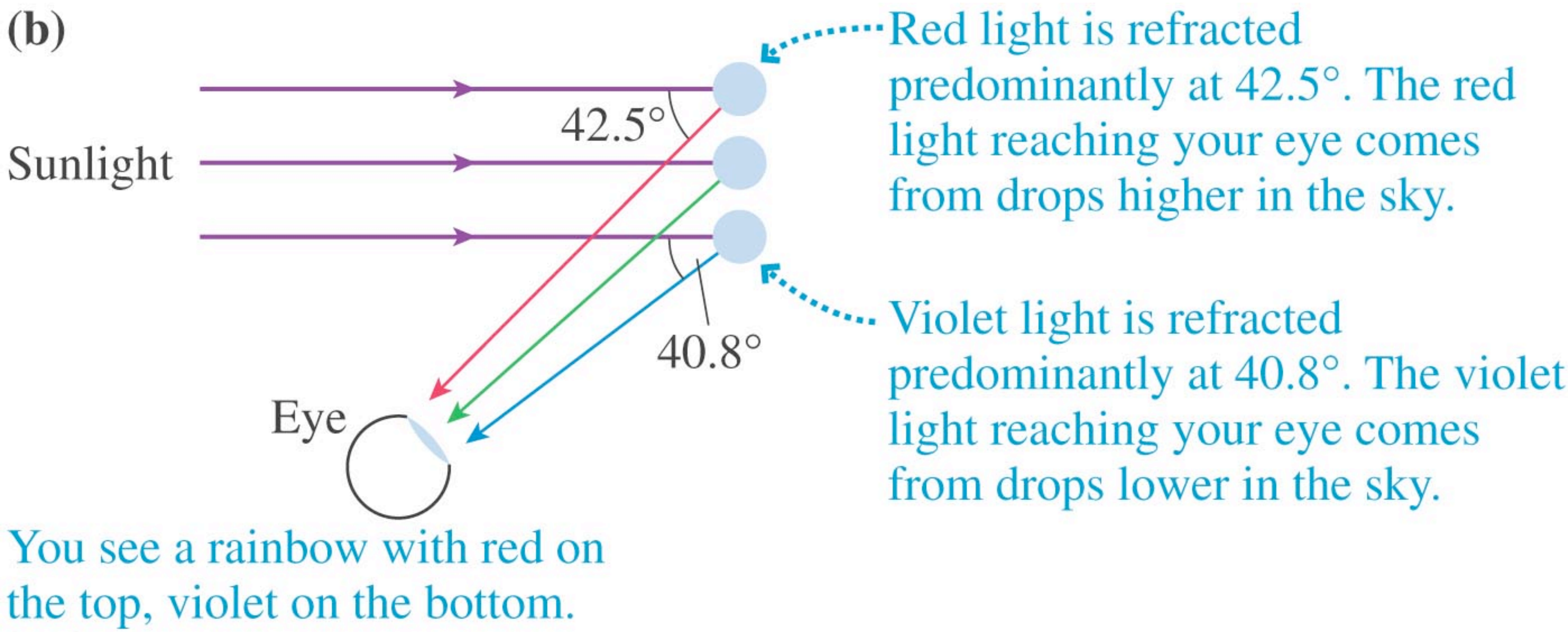



(a)
A second prism can combine the colors back into white light.
White light
White light
A prism disperses white light into colors.


(b)
The second prism does not change pure colors.
White light
Green light
An aperture selects a green ray of light.

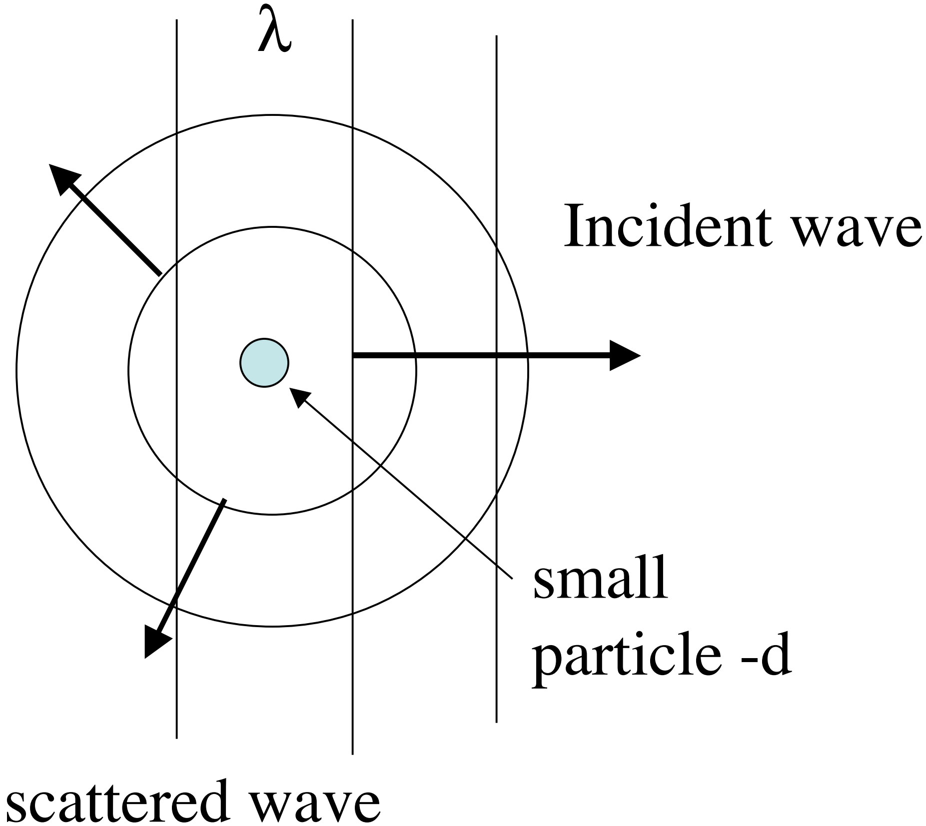
Examples of dispersive refraction - Rainbow





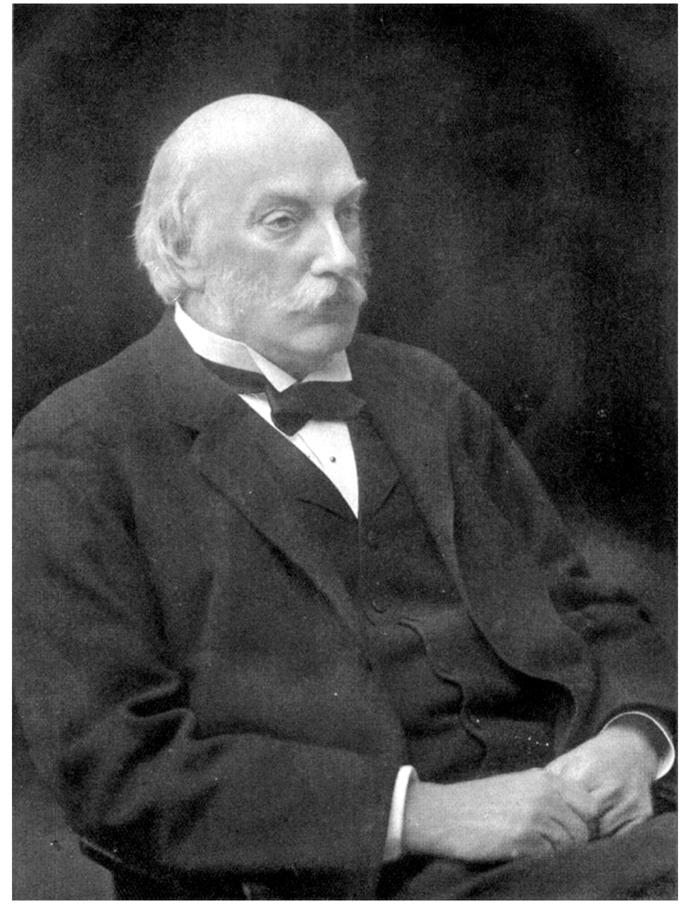
(b)
Sunlight
42.5°
40.8°
Eye
Red light is refracted predominantly at 42.5°. The red light reaching your eye comes from drops higher in the sky.
Violet light is refracted predominantly at 40.8°. The violet light reaching your eye comes from drops lower in the sky.
You see a rainbow with red on the top, violet on the bottom.

Rayleigh Scattering



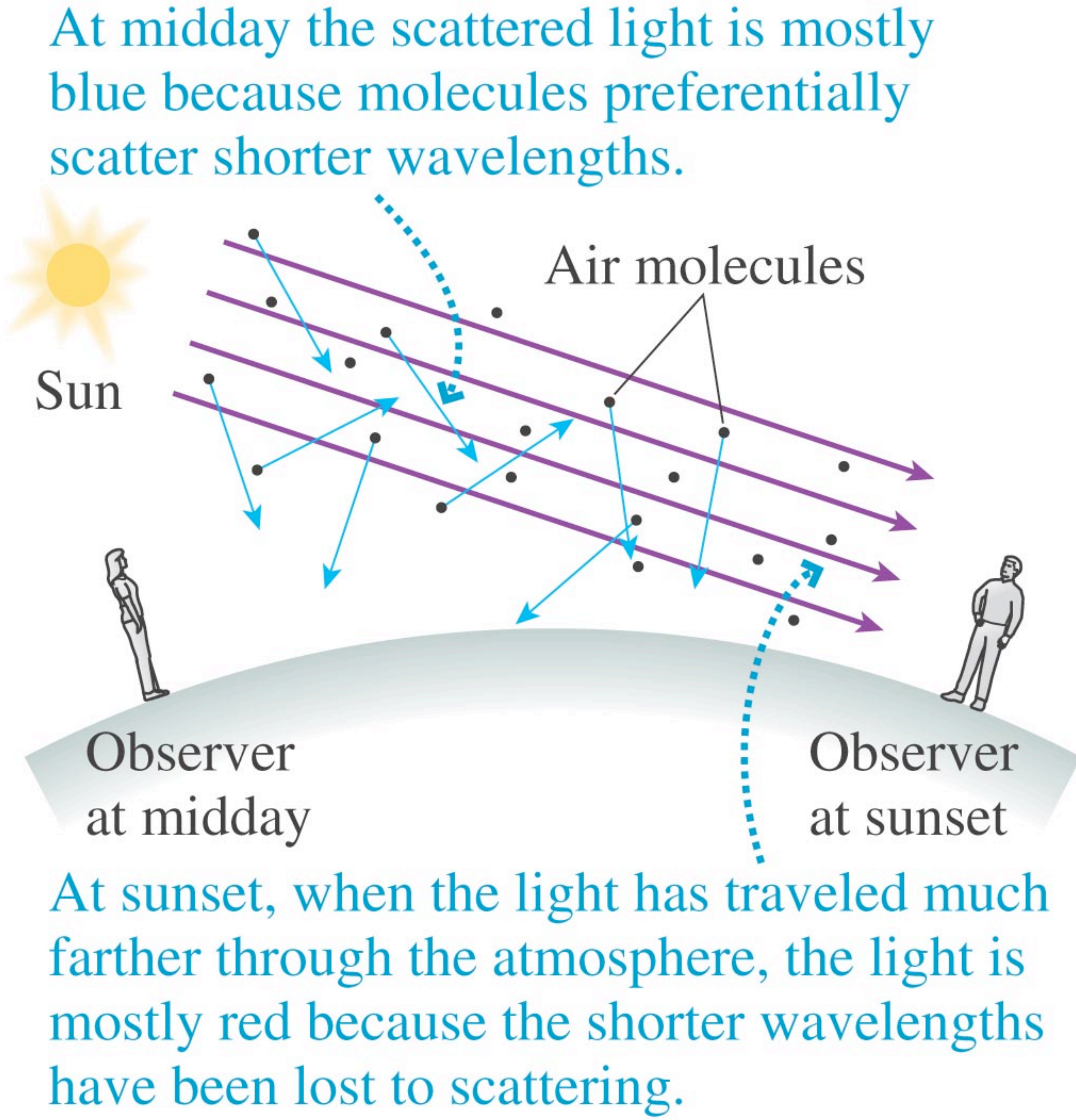
$$I \propto \frac{d^6}{\lambda^4 R^2}$$

scattered intensity is higher for shorter wavelengths



John William Strutt
3rd Baron Rayleigh

Wikimedia commons

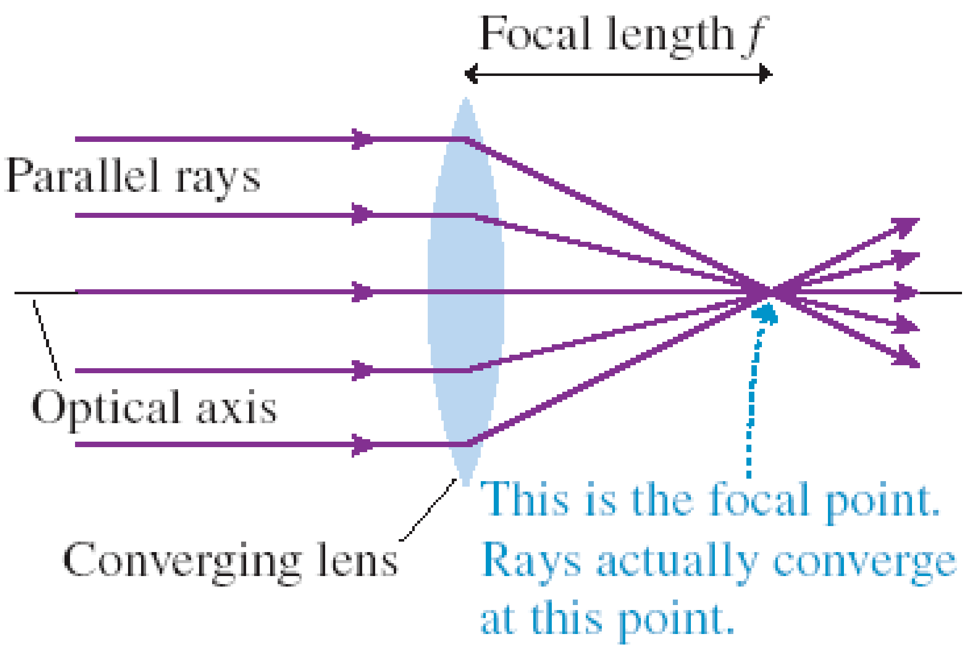


At midday the scattered light is mostly blue because molecules preferentially scatter shorter wavelengths.
Air molecules
Sun
Observer at midday
Observer at sunset
At sunset, when the light has traveled much farther through the atmosphere, the light is mostly red because the shorter wavelengths have been lost to scattering.

Lenses

Thin Lenses: Ray Tracing

FIGURE 23.34 The focal point and focal length of converging and diverging lenses.



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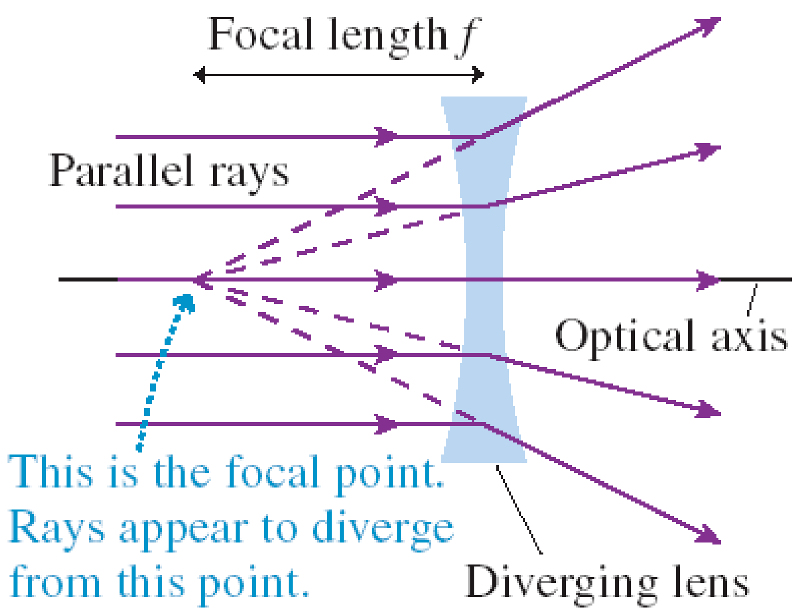
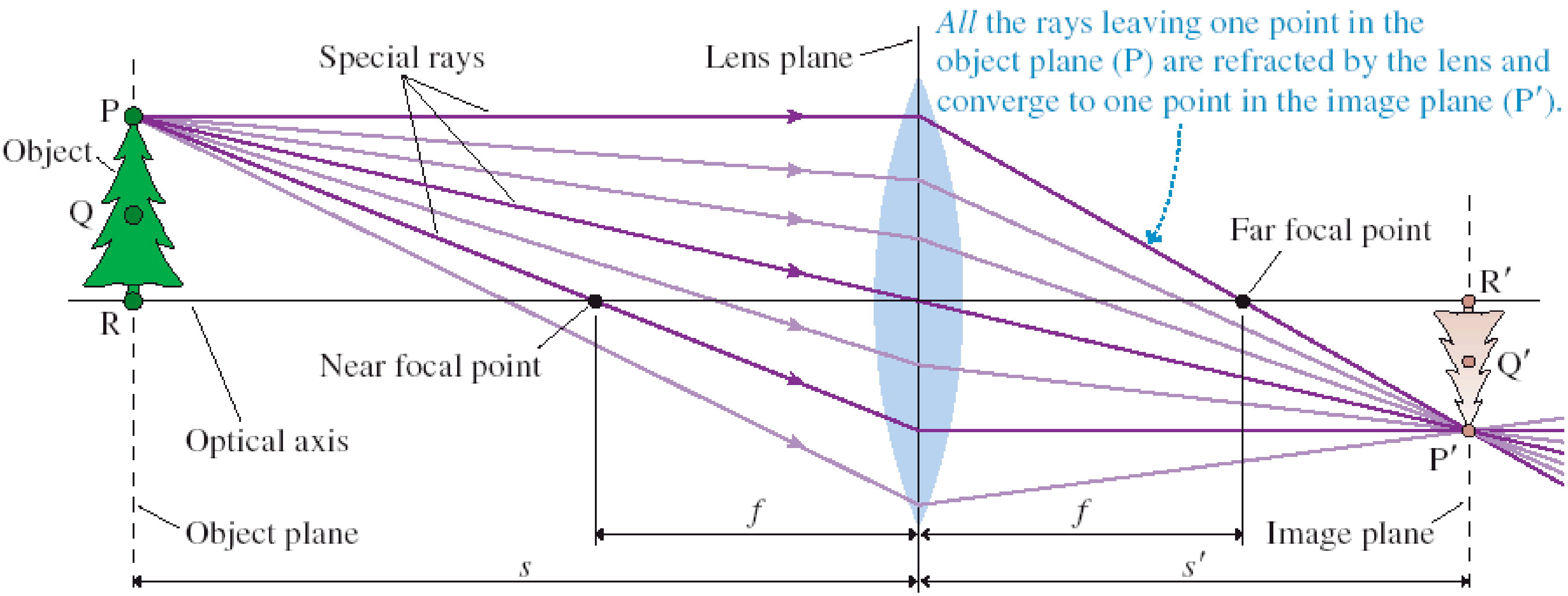


Thin Lenses: Ray Tracing

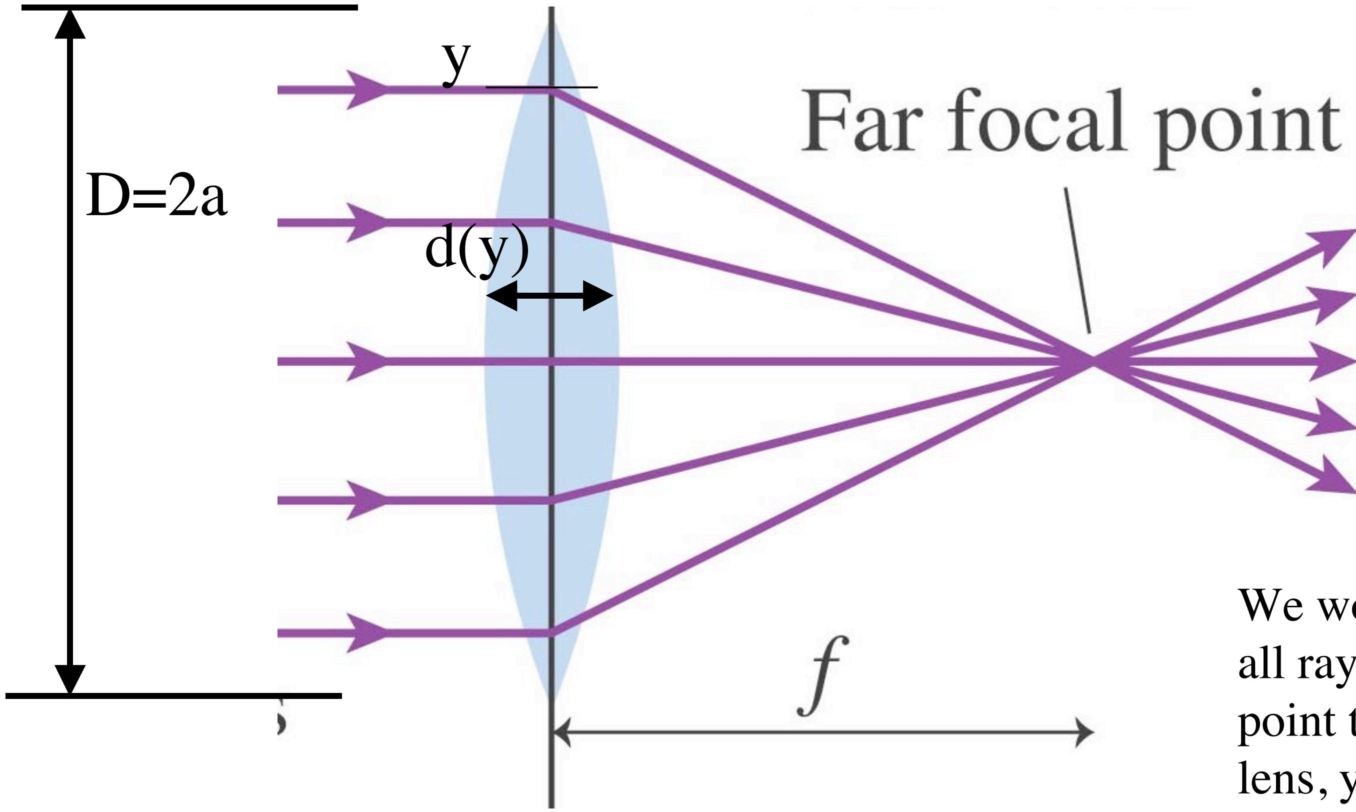
FIGURE 23.36 Rays from an object point P are refracted by the lens and converge to a real image at point P′.



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(thin-lens equation)}$$

Real Image

Thin lens approximation $d << D, f$



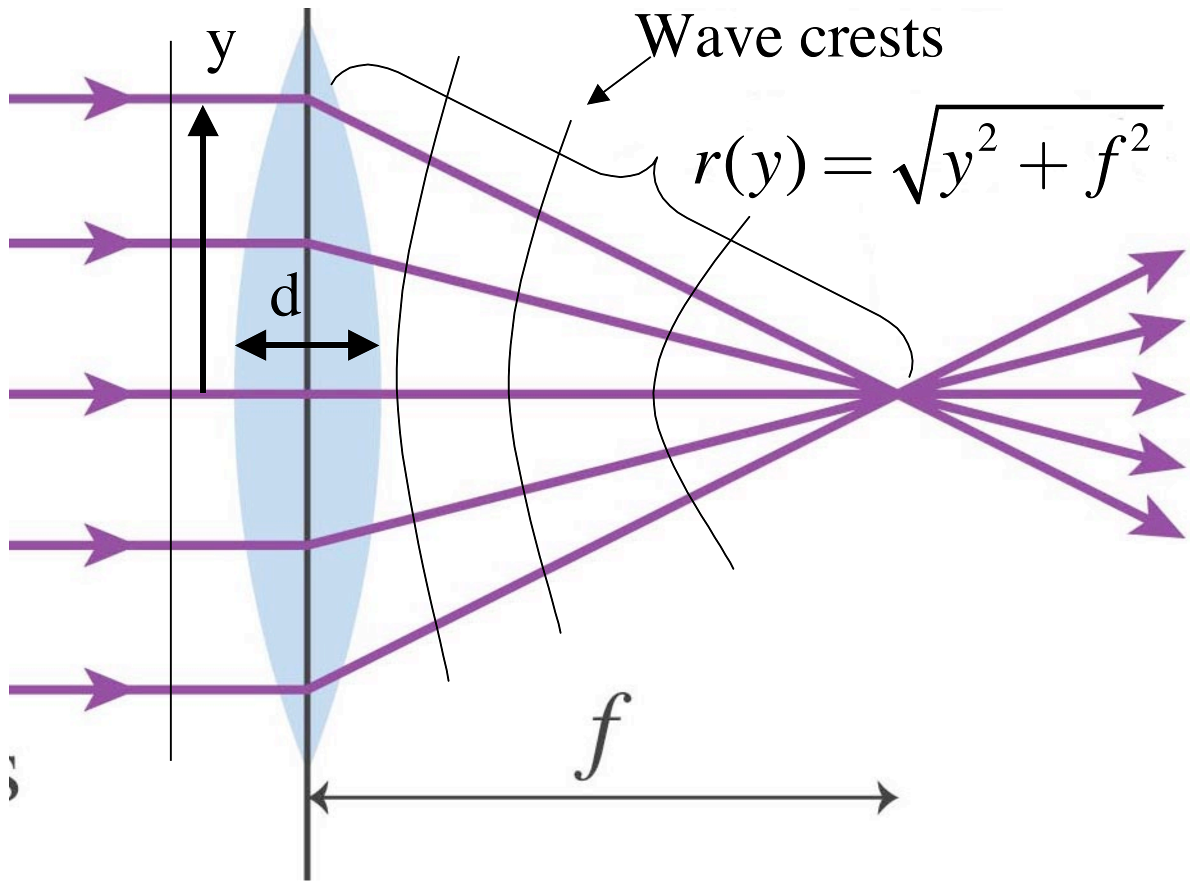
We would like to show that all rays, independent of the point they pass through the lens, y, focus to the same point f.

Lens has parabolic thickness

$$d(y) = \frac{a^2 - y^2}{2L}$$

Determines focal length

What is the phase of a wave arriving at the focus?



$k = 2\pi / \lambda_{vac}$

Contribution from lens

Wave phase

$$\phi = k\left[(n-1)d(y) + r(y)\right]$$

Contribution from region between lens and focus

Some Math

Phase $$\phi = k\left[(n-1)d(y) + r(y)\right]$$

Recall thickness of lens $$d(y) = \frac{a^2 - y^2}{2L}$$

Approximate $$r(y) = \sqrt{y^2 + f^2} \simeq f + \frac{y^2}{2f}$$

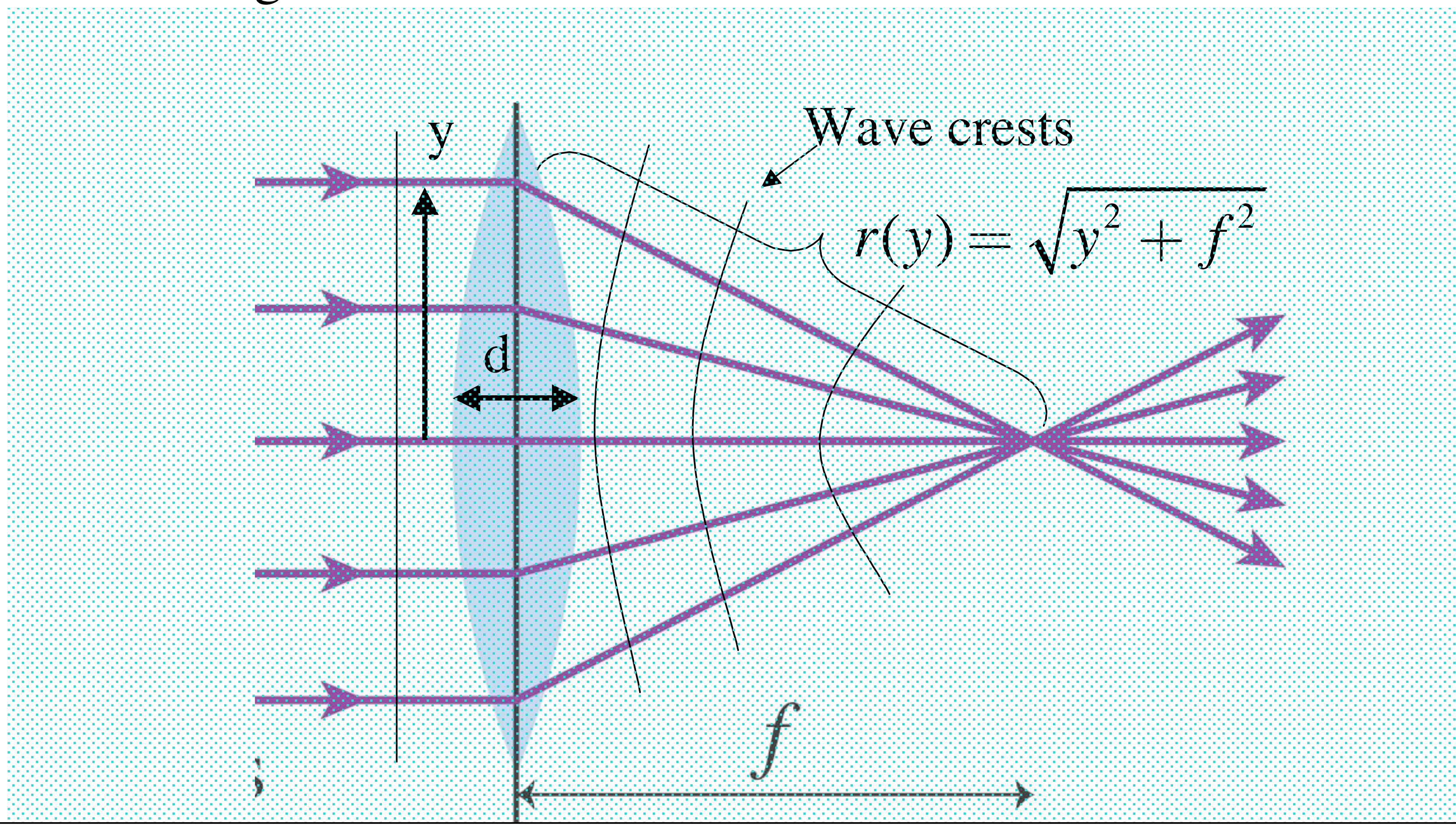
$$y << f$$

Phase is independent of ray (y) if $$\frac{-(n-1)y^2}{2L} + \frac{y^2}{2f} = 0$$

Focal length determined by curvature of lens and index of refraction

$$\frac{1}{f} = \frac{(n-1)}{L}$$

What changes when the lens is immersed in another medium ?

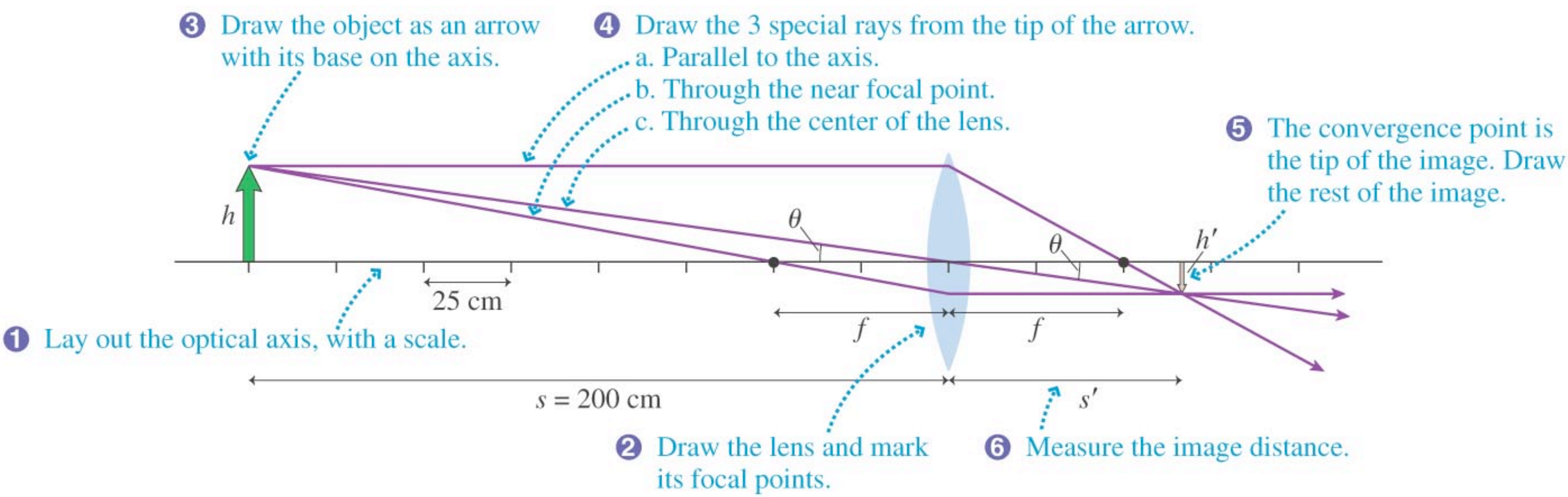


$k = 2\pi n_{medium} / \lambda_{vac}$ Contribution from lens

Wave phase $\phi = k\left[(n_{lens} - n_{medium})d(y) + r(y)\right]$

$$\frac{1}{f} = \frac{(n_{lens} - n_{medium})}{L}$$

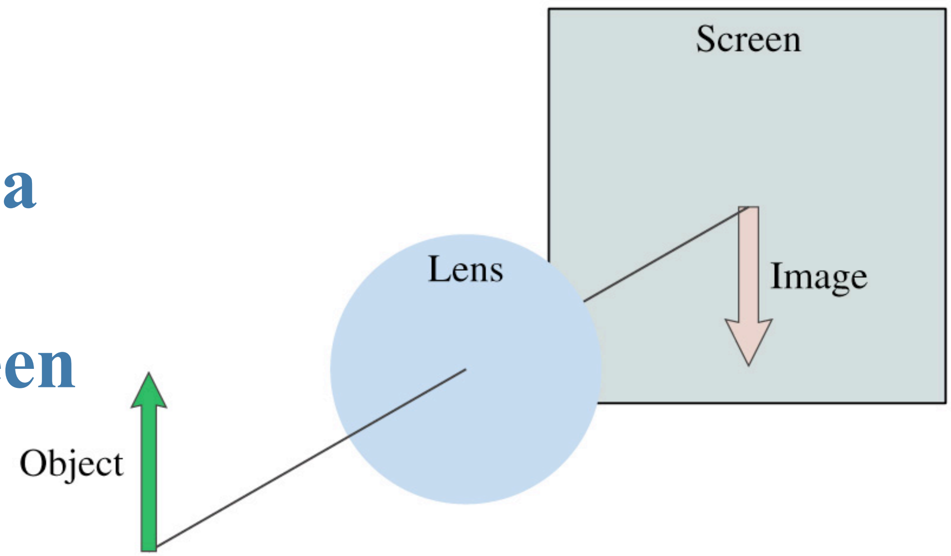
Graphically locating an image and determining it's size



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(thin-lens equation)}$$

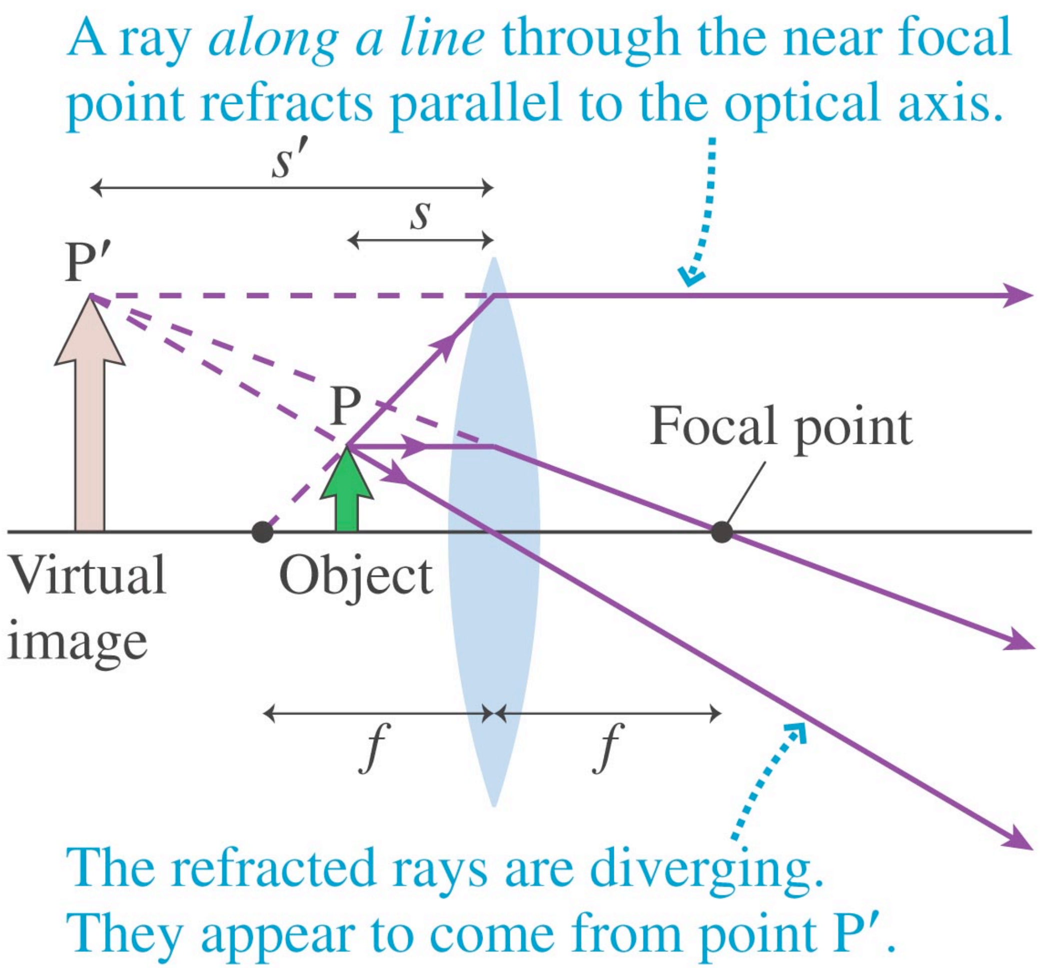
$$\frac{h'}{h} = -\frac{s'}{s} = m$$

A lens produces a sharply-focused, inverted image on a screen. What will you see on the screen if the lens is removed?



A. The image will be inverted and blurry.
B. The image will be as it was, but much dimmer.
C. There will be no image at all.
D. The image will be right-side-up and sharp.
E. The image will be right-side-up and blurry.

Suppose object is closer than focal point to lens

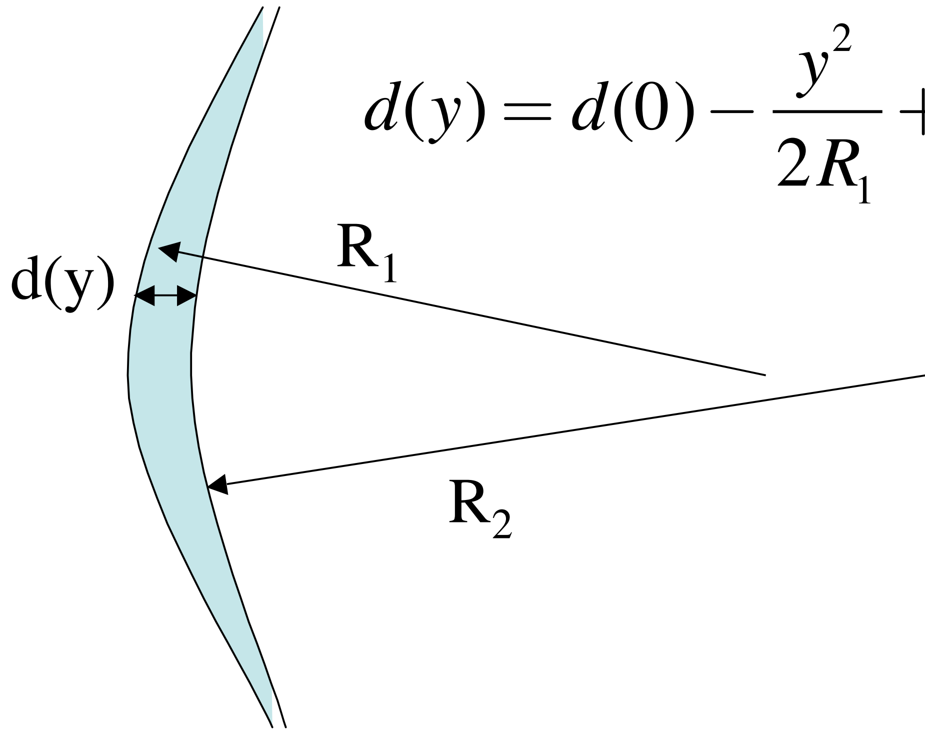


Virtual mage located at $s' < 0$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{(thin-lens equation)}$$

$$\frac{h'}{h} = -\frac{s'}{s} = m$$

Lens Maker Formula: two surfaces defined by two radii of curvature



$$d(y) = d(0) - \frac{y^2}{2R_1} + \frac{y^2}{2R_2}$$

Compare with

$$d(y) = \frac{a^2 - y^2}{2L}$$

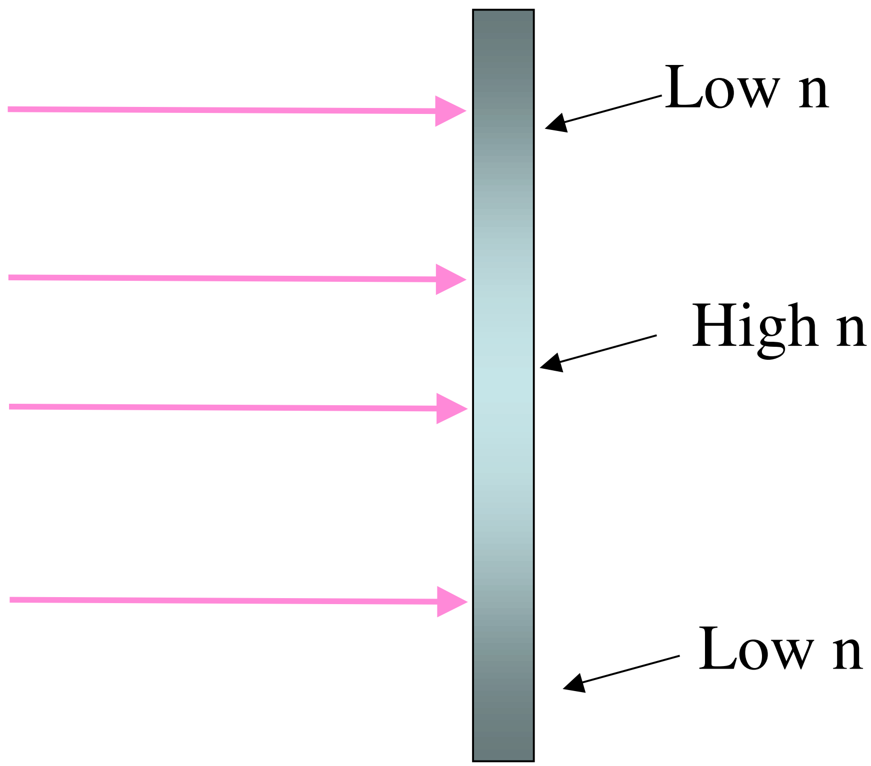
The same coefficient of y^2 if

$$\frac{1}{2L} = \frac{1}{2R_1} - \frac{1}{2R_2}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Works for both converging and diverging lens

A lens is made of a material with two flat parallel surfaces. The material has a non-uniform index of refraction



Will the rays

a) Converge
b) Diverge
c) Go straight
d) Spiral
e) Become so frustrated that the fall down to the ground