Erratum to "Feasible sequential quadratic programming for finely dicretized problems from SIP"

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Abstract

Two errors are pointed out in the convergence analysis in the quoted paper [Semi-Infinite Programming, Kluwer Academic Publishers, 1998, pp. 159–193]. Correct proofs are provided.

Below, two errors are pointed out in the convergence analysis in the quoted paper [1]. Correct proofs are provided. Note that these proofs only apply when $d^1(x,\hat{\Xi})$ is defined to be the solution of $QP^1(x,\hat{\Xi})$.

In the proof of Lemma 3.5(iii), Lemmas 3.1 and 3.2 are both applied without first showing that $\Xi_{\text{act}}(x^*) \subseteq \Xi^*$. We offer here an alternative proof which does not rely on establishing this fact.

Lemma 3.5(iii)**:** $t_k d_k \to 0$.

Proof. In view of non-positivity of the optimal values for the QPs defining d_k^0 and d_k^1, the inequalities

$$\langle \nabla f(x_k), d_k^0 \rangle \leq -\frac{1}{2}\langle d_k^0, H_k d_k^0 \rangle,$$
$$\langle \nabla f(x_k), d_k^1 \rangle \leq -\frac{1}{2}\langle d_k^1, d_k^1 \rangle$$

hold. Thus,

$$\begin{aligned}\langle \nabla f(x_k), d_k \rangle &= (1-\rho_k)\langle \nabla f(x_k), d_k^0 \rangle + \rho_k \langle \nabla f(x_k), d_k^1 \rangle \\ &\leq -\frac{(1-\rho_k)}{2}\langle d_k^0, H_k d_k^0 \rangle - \frac{\rho_k}{2}\langle d_k^1, d_k^1 \rangle.\end{aligned}$$

In view of the line search criterion

$$\begin{aligned}f(x_k) - f(x_{k+1}) &\geq -\alpha t_k \langle \nabla f(x_k), d_k \rangle \\ &\geq \frac{\alpha t_k (1-\rho_k)}{2}\langle d_k^0, H_k d_k^0 \rangle + \frac{\alpha t_k \rho_k}{2}\langle d_k^1, d_k^1 \rangle \\ &\geq 0.\end{aligned}$$

Since we have established that the sequence $\{f(x_k)\}$ converges, and since both terms on the right-hand side of the second line are positive, they must both converge to zero. Further, note that the inequalities still hold if the first term on the right-hand side of the second line is multiplied by $t_k(1-\rho_k) \in [0,1]$ and the second term is multiplied by $t_k \rho_k \in [0,1]$ (since both terms are positive). As H_k is positive definite for all k and bounded away from singularity, this gives

$$t_k(1-\rho_k)d_k^0 \to 0,$$

$$t_k \rho_k d_k^1 \to 0.$$

Adding these shows $t_k d_k \to 0$. □

In the proof of Proposition 3.7 in the appendix, in the second paragraph, the logic used to establish that $d^{0,*}$ and $d^{1,*}$ are both non-zero is flawed. In particular, again it has not been established that $\Xi_{\text{act}}(x^*) \subseteq \Xi'$, hence Lemmas 3.1 and 3.2 may not be applied. Note that under the contradiction assumption of Proposition 3.7, $v^{0,*} + v^{1,*} > 0$. Thus, in view of Lemma 3.6(i) and (ii), we cannot have *both* $d^{0,*} = 0$ and $d^{1,*} = 0$, i.e. at least one must be non-zero. This, coupled with the following claim, gives the result we need.

Claim: Given $x \in X$, $H = H^T > 0$, and $\Xi' \subseteq \Xi$, $d^0(x, H, \Xi') = 0$ if, and only if, $d^1(x, \Xi') = 0$.

Proof. Suppose $d^0 = d^0(x, \Xi') = 0$ and let λ_ξ^0, $\xi \in \Xi'$, denote the multipliers from $QP^0(x, H, \Xi')$. From the optimality conditions (2.2) for $QP^0(x, H, \Xi')$ and (2.3) for $QP^1(x, \Xi')$ it is not difficult to see that $(d^1, \gamma) = (0, 0)$ is a

KKT point for $QP^1(x, \Xi')$ with multipliers

$$\mu^1 = \frac{1}{1 + \sum_{\xi \in \Xi'} \lambda_\xi^0},$$
$$\lambda_\xi^1 = \frac{\lambda_\xi^0}{1 + \sum_{\xi \in \Xi'} \lambda_\xi^0}, \quad \xi \in \Xi'.$$

Uniqueness of such points establishes $d^1(x, \Xi') = 0$.

Now suppose $d^1 = d^1(x, \Xi') = 0$, thus $\gamma(x, \Xi') = 0$, and let μ^1 and λ_ξ^1, $\xi \in \Xi'$, denote the multipliers from $QP^1(x, \Xi')$. As $(d^1, \gamma) = (0, 0)$, it follows from the constraints in $QP^1(x, \Xi')$ that $\Xi^1 \subseteq \Xi_{\text{act}}(x)$, where $\Xi^1 \subseteq \Xi'$ is the active set from $QP^1(x, \Xi')$. This implies $\mu^1 > 0$, otherwise the first equation in the optimality conditions (2.3) would violate Assumption 2. It then follows from the optimality conditions (2.2) and (2.3) that $d^0 = 0$ is KKT for $QP^0(x, H, \Xi')$ with multipliers

$$\lambda_\xi^0 = \frac{\lambda_\xi^1}{\mu^1}, \quad \xi \in \Xi'.$$

Uniqueness of such points establishes $d^0(x, H, \Xi') = 0$. □

References

[1] C. T. Lawrence and A. L. Tits. Feasible sequential quadratic programming for finely discretized problems from SIP. In R. Reemtsen and J.-J. Rückmann, editors, *Semi-infinite Programming*, pages 159–193. Kluwer Academic Publishers B.V., 1998. In the series Nonconvex Optimization and its Applications.