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LAST NAME: **First Name:**

Grade:

ENEE 222 FALL 2021
FINAL EXAMINATION (SECOND SESSION)

- Closed book, no notes, no calculator.
- Show your work clearly, justifying your answers where appropriate.
- Enter your name (LAST followed by first) on all sheets.
- **Do not** detach any sheets. Use this sheet for rough work.
- Limit your answer to any single problem to the space provided. If you need more space, continue on the **back side of the same sheet.**
- The problem statements should be self-explanatory. If you need a clarification, come to the front and submit **your query in writing.**
- At the end of the exam, make sure your name is on all five sheets. Please leave your booklet on your desk and exit the room quietly.

	Periodic Time-Domain Signal	*Fourier Series Coefficients*
	$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\Omega_0 t}$	$X_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\Omega_0 t}dt$
	$x^*(t)$	X^*_{-k}
	$x(-t)$	X_{-k}
	$x(t-T)$	$e^{-jk\Omega_0 T}X_k$
$(K \in \mathbf{Z})$	$x(t)e^{jK\Omega_0 t}$	X_{k-K}
$(\beta > 0)$	$x(\beta t)$	X_k

Common Trigonometric Values

$\cos(\pi/6) = \sin(\pi/3) = \sqrt{3}/2;$ $\cos(\pi/4) = \sin(\pi/4) = \sqrt{2}/2;$ $\cos(\pi/3) = \sin(\pi/6) = 1/2$

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PROBLEM 1 (15 pts.)

With t in seconds, let

$$x(t) \;=\; -1.7 \,+\, 5.5\cos(36\pi t - 1.9) \,+\, 2.3\cos(54\pi t - 0.8) \,+\, 1.4\cos(90\pi t + 2.1)$$

(i) (2 pts.) Determine the fundamental period T_0 of $x(t)$.

(ii) (4 pts.) Define (i.e., give the numerical values of) a scalar `c` and a vector `b` having the *smallest possible length*, such that

```
c =                ;
b = [                                ].'  ;
z = c*ifft(b,400) ;
x = real(z)       ;
```

generates a vector `x` consisting of 400 samples of $x(t)$ uniformly spaced in $[0, T_0)$.

(iii) (5 pts.) If `b` and `c` are as specified in **(ii)** above, the code

```
b_new = [-j*3.2 ; 0 ; conj(b)]    ;    % CONJ(.) : COMPLEX CONJUGATE
z = c*ifft(b_new,200)             ;
s = imag(z)                       ;    % IMAG(.) : IMAGINARY PART OF COMPLEX NUMBER
```

produces 200 samples, uniformly spaced in $[0, T_0)$, of a periodic signal $s(t)$ with the same fundamental period (i.e., T_0). Assuming $s(t)$ has no harmonics higher than the 100th, write an equation for $s(t)$.

(iv) (4 pts.) Suppose that $x(t)$ (*not* $s(t)$) is the input to an ideal *bandpass* filter which allows all frequencies in the interval $(f_c, 2f_c)$ to pass through (with unit gain and zero delay), and blocks all other frequencies. If the output of the filter is a single nontrivial sinusoid, what are *all* the possible values of f_c, in Hz?

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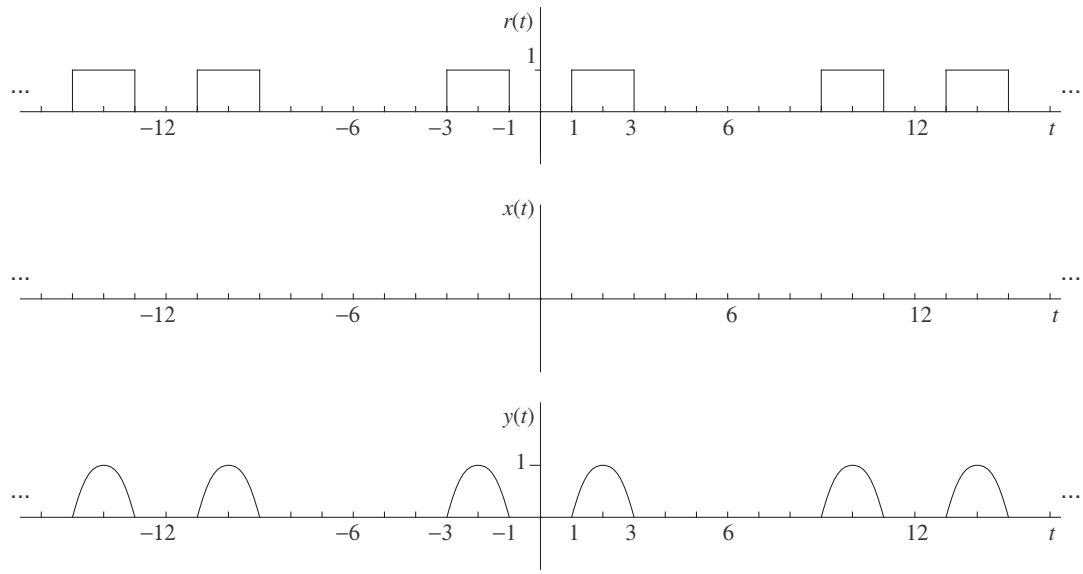
PROBLEM 2 (15 pts.)

The signal $r(t)$ shown below is periodic with period $T_0 = \mathbf{12}$ and has complex Fourier series expansion

$$r(t) \;=\; \sum_{k=-\infty}^{\infty} R_k e^{jk\Omega_0 t} \;,$$

where Ω_0 is the fundamental angular frequency.



(i) (2 pts.) What is the value of R_0?

Let $x(t)$ be periodic with the same period as $r(t)$, and such that

$$X_0 \;=\; -1/3 \qquad \text{and} \qquad X_k \;=\; 2R_k \cos(k\pi/6) \quad (k \neq 0)$$

(ii) (5 pts.) Express $x(t)$ in terms of $r(t)$.

(iii) (2 pts.) Sketch $x(t)$ on the blank axes provided, clearly marking the scale on the vertical axis. Verify that your graph agrees with the value of X_0 given above.

(iv) (3 pts.) Express the real-valued periodic signal $y(t)$ (bottom graph) in terms of $r(t)$. *The curved sections of the graph are sinusoidal half-cycles.*

(v) (3 pts.) Express the kth Fourier series coefficient Y_k of $y(t)$ in terms of of R_k's.

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PROBLEM 3 (15 pts.)

Consider the FIR filter with impulse response

$$h[n] \;=\; \delta[n] - \delta[n-1] + 3\delta[n-2] - 2\delta[n-3] + 3\delta[n-4] - \delta[n-5] + \delta[n-6]$$

(i) (4 pts.) Express the filter's frequency response in the form

$$H(e^{j\omega}) \;=\; e^{-j\omega\alpha} F(\omega) \;,$$

where $F(\omega)$ is real-valued.

(ii) (5 pts.) If the input to the filter is given at all times n by

$$x[n] \;=\; 2^{-n} + 3(-1)^n \;,$$

determine the output $y[n]$ at all times n, simplifying your answer as much as possible.

(iii) (3 pts.) Suppose the filter input is periodic with period $L = 4$. In the code segment below, vector `x` consists of input samples at times $0:3$. How should vector `b` be defined (numerically) in order for `y` to equal the corresponding output samples (also at times $0:3$)?

```
B = fft(b)     ;     % You may write your answer here:  b = [                    ]
X = fft(x)     ;
Y = B.*X       ;
y = ifft(Y)    ;
```

(iv) (3 pts.) Show that the filter can be implemented as a series connection of two FIR filters with impulse responses

$$\begin{aligned} h^{(1)}[n] &= \delta[n] + \delta[n-2] \\ h^{(2)}[n] &= \delta[n] + r\delta[n-1] + s\delta[n-2] + r\delta[n-3] + \delta[n-4] \end{aligned}$$

and determine the values of r and s.

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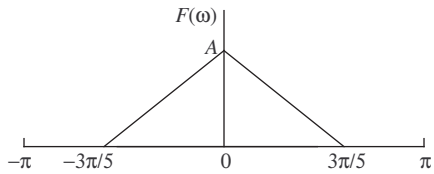
PROBLEM 4 (15 pts.)

Consider a filter whose frequency response is given by

$$H(e^{j\omega}) \;=\; \boldsymbol{j}e^{-j3\omega}F(\omega)\ ,$$

where $F(\omega)$ is as shown below.



(i) (2 pts.) Express the filter's magnitude response $|H(e^{j\omega})|$ in terms of $F(\omega)$.

Suppose the filter input

$$x[n] \;=\; \cos\left(\frac{\pi n}{4}\right)$$

produces the output

$$y[n] \;=\; 2\cos\left(\frac{\pi n}{4} + \phi\right)$$

(ii) (4 pts.) Determine the value of A, expressing it as a ratio of two integers.

(iii) (3 pts.) Determine the value of ϕ (phase of $y[n]$).

(iv) (3 pts.) Give an example of an input sequence $x'[n]$, having the *same period* as $x[n]$, such that resulting output $y'[n]$ also equals $y[n]$ as given above.

(v) (3 pts.) Suppose $y[n]$ (as given above) is applied to a second filter with impulse response

$$h[n] \;=\; \delta[n] + b\delta[n-1] + \delta[n-2]$$

Determine the value of b such that resulting output (call it $v[n]$) equals zero for all n.

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PROBLEM 5 (15 pts.)

Let

```
h = [ 2 3 3 2 ].' ;
s = [ 5 -4  3  -2  1 ].'    ;
H = fft(h,8)     ;
S = fft(s,8)     ;
C = H.*S         ;
c = ifft(C)      ;
```

(i) (6 pts.) Determine the vector `c`.

(ii) (3 pts.) Based on the symmetry in `s`, explain how the result would differ if `s` were replaced by its time-reverse

```
s_rev = [ 1 -2  3 -4  5 ].'    ;
```

(iii) (6 pts.) Using the answers to **(i)** and **(ii)** above, determine the vector `y` obtained by

```
h = [ 2 3 3 2 ].' ;
x = [ 5 -4  3 -2  0  2 -3  4 -5 ].' ;
G = fft(h,13) ;   % 13, not 12
X = fft(x,13) ;
Y = G.*X      ;
y = ifft(Y)   ;
```

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