ENEE626. Midterm examination, 10/23/2008 **Instructor:** A. Barg

All answers should be accompanied with proofs or sufficient explanation. Intermediate calculations should be shown.

10 points for each of the questions.

(a) Let the binary linear code $\mathcal{C}$ be generated by the next matrix.

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0 0 0 1 1 1 0 0 1
0 0 1 0 0 0 1 1 0
1 1 0 0 0 1 1 0 1
```

Find its dimension. Does $\mathcal{C}$ correct all single errors? Double errors? If not, find a noncorrectable double error and its coset leader.

(b) The finite field $\mathbb{F}_9$ can be constructed by adding a root α of $f(x) = x^2 - x - 1$ to $\mathbb{F}_3$. Complete the following table

Power of α	vector in the basis $1, \alpha$	vector in the basis α, α^3	power of α^3
$-\infty$ 0 1 $\vdots$	$\vdots$	$\vdots$	$\vdots$

(c) Which of the following pairs of elements of $\mathbb{F}_9$

$$(1,\alpha), (1,\alpha^2), (1,\alpha^3), (1,\alpha^4), (1,\alpha^5), (1,\alpha^6), (1,\alpha^7), (1,\alpha^8), (\alpha,\alpha^2), (\alpha,\alpha^3), (\alpha,\alpha^4)$$
$$(\alpha,\alpha^5), (\alpha,\alpha^6), (\alpha,\alpha^7), (\alpha^2,\alpha^3), (\alpha^2,\alpha^6), (\alpha^3,\alpha^4)$$

form a basis of $\mathbb{F}_9$ over $\mathbb{F}_3$?

(d) Let $\mathscr{P} = (1, \alpha, \alpha^2, \dots, \alpha^7)$ and let $\mathcal{R}$ be an $[8, 3]$ RS code over $\mathbb{F}_9$ with the defining set $\mathscr{P}$. Find a codeword of weight equal to the minimum distance of $\mathcal{R}$.

(e) What is the coset leader of the vector $\boldsymbol{y} = (1, \alpha, \alpha^4, \alpha^3, \alpha^4, \alpha^5, \alpha^2, \alpha^7)$ with respect to the code $\mathcal{R}$?

(f) Decode the vector $\boldsymbol{y} = (0, \alpha^7, \alpha^4, \alpha^5, 1, \alpha^6, \alpha^3, \alpha^2)$ with the code $\mathcal{R}$.

(g) Let $x \in \mathbb{F}_{p^m}$. Prove that $z(x) = x + x^p + x^{p^2} \cdots + x^{p^{m-1}} \in \mathbb{F}_p$ and that the number of different x for which $z(x) = 0$ equals p^{m-1}.