a) \( Y \sim \text{uniform } [0,1] \)
\[
P(Y \leq y) = P \left( \frac{1}{u+1} \leq y \right) 
= P \left( u+1 \geq \frac{1}{y} \right) 
= P \left( u \geq \frac{1}{y} - 1 \right) 

F_Y(y) = \begin{cases} 
1 - \left( \frac{1}{y} - 1 \right) & ; 0 \leq \frac{1}{y} - 1 \leq 1 \Rightarrow \frac{1}{2} \leq y \leq 1 \\
0 & ; \frac{1}{y} - 1 \geq 1 \Rightarrow y \leq \frac{1}{2} \\
1 & ; \frac{1}{y} - 1 \leq 0 \Rightarrow y \geq 1 
\end{cases}

f_Y(y) = \begin{cases} 
0 & ; y \leq \frac{1}{2} \\
2 - \frac{1}{y} & ; \frac{1}{2} < y \leq 1 \\
1 & ; y \geq 1 
\end{cases}

b) \( Y = \log (u+1) \)
\[
P(Y \leq y) = P(\log (u+1) \leq y) 
= P( u+1 \leq e^y) 
= P( u \leq e^{y-1}) 

F_Y(y) = \begin{cases} 
0 & ; e^{y-1} \leq 0 \Rightarrow y \leq 0 \\
e^{y-1} & ; 0 \leq e^{y-1} \leq 1 \Rightarrow 0 \leq y \leq \ln(2) \\
1 & ; e^{y-1} \geq 1 \Rightarrow y \geq \ln(2) 
\end{cases}

f_Y(y) = \begin{cases} 
0 & ; y \leq 0 \\
e^{y-1} & ; 0 \leq y \leq \ln(2) \\
0 & ; y \geq \ln(2) 
\end{cases}
\[
Y = X + b \\
F_Y(y) = P(Y \leq y) \\
= P(X + b \leq y) \\
= P(X \leq y - b) \\
= F_X(y - b) \\
\]

\[z = ax\]

\[a > 0 \quad F_z(z) = P(Z \leq z) = P(aX \leq z) = P(X \leq \frac{z}{a}) = F_X(\frac{z}{a})\]

\[a = 0 \quad z = 0 \quad F_z(z) = \begin{cases} 0 & ; 3 < 0 \\ 1 & ; 3 \geq 0 \end{cases}\]

\[a < 0 \quad F_z(z) = P(Z \leq z) = P(aX \leq z) = P(X \geq \frac{z}{a}) = 1 - P(X < \frac{z}{a}) = 1 - F_X(\frac{z}{a})_{\text{continuous}}\]

\[W = aX + b\]

\[a > 0 \quad F_W(w) = P(W \leq w) = P(aX + b \leq w) = P(X \leq \frac{w - b}{a}) = F_X(\frac{w - b}{a})\]

\[a = 0 \quad W = b \quad F_W(w) = \begin{cases} 0 & ; w < b \\ 1 & ; w \geq b \end{cases}\]

\[a < 0 \quad F_W(w) = P(W \leq w) = P(aX + b \leq w) = P(X \geq \frac{w - b}{a}) = 1 - P(X < \frac{w - b}{a}) = 1 - F_X(\frac{w - b}{a})_{\text{continuous}}\]

\[f(x) = \lambda e^{-\lambda x} \quad g(x) = \mu e^{-\mu x}\]

\[P(X < Y) = \int_0^\infty P(Y > x \mid X = x) f_X(x) \, dx \]

\[= \int_0^\infty P(Y > x) f_X(x) \, dx = \int_0^\infty \left[ \int_x^\infty f_Y(y) \, dy \right] f_X(x) \, dx \]

\[= \int_0^\infty \int_x^\infty f_Y(y) \, dy \, f_X(x) \, dx = \int_0^\infty \left[ \int_0^\infty g(y) \, dy \right] f_X(x) \, dx \]

\[= \int_0^\infty (1 - e^{-\lambda x}) f_X(x) \, dx = \int_0^\infty (1 - G(x)) f_X(x) \, dx \]

\[= \int_0^\infty e^{-\lambda x} \mu e^{-\mu x} \, dx = \int_0^\infty f_X(x) (1 - G(x)) \, dx \]

\[P(X < Y) = \int_0^\infty \lambda e^{-\lambda x} (1 - (1 - e^{-\mu x})) \, dx \]

\[= \int_0^\infty \lambda e^{-\lambda x} e^{-\mu x} \, dx \]

\[= \lambda \left[ \frac{e^{-(\lambda + \mu)x}}{-\lambda + \mu} \right]_0^\infty \]

\[= \frac{-\lambda}{\lambda + \mu} (0 - 1) = \frac{1}{\lambda + \mu}\]
\[ Y : \text{lifetime of 100W bulb} \quad \mu = \frac{1}{100} \]
\[ X : \quad 60W \quad \lambda = \frac{1}{200} \]

\[ P(X \leq Y) = \frac{\lambda}{\lambda + \mu} = \frac{\frac{1}{200}}{\frac{1}{200} + \frac{1}{100}} = \frac{3}{4} \]

\text{p. 279, 10 \(a\)}

\[ Z = |X - Y| \]

\[ P(Z \leq 3) = P(1X - Y | 1 \leq 3) = P(-3 \leq X - Y \leq 3) = P(Y \leq X + 3) \cap Y \geq X - 3) \]

\[ F_z(3) = \begin{cases} 1 & 3 \leq 1 \\ 0 & 3 > 1 \end{cases} \]

\[ f_z(3) = \begin{cases} 2(1 - 3) & 3 \leq 1 \\ 0 & 3 > 1 \end{cases} \]

\[ E[|X - Y|] = \int_0^3 f_z(3) \, d3 = 2 \left[ \frac{3^2}{2} \right]_0^3 = 2 \left[ \frac{27}{2} \right]_0^3 = 2 \left[ \frac{27}{2} \right] = 2 \times \frac{27}{2} = 27 \]

\text{p. 279, 10 \(b\)}

\[ Z = \max(X, Y) \]

\[ P(Z \leq 3) = P(\max(X, Y) \leq 3) = P(X \leq 3) \cap Y \leq 3) = P(X \leq 3) P(Y \leq 3) \]

\[ F_z(3) = \begin{cases} 0 & 3 \leq 1 \\ 1 & 3 > 1 \end{cases} \]

\[ f_z(3) = \begin{cases} 0 & 3 \leq 1 \\ \frac{2}{3} & 3 \leq 1 \end{cases} \]

\[ E[\max(X, Y)] = \int_0^3 f_z(3) \, d3 = \int_0^3 \frac{2}{3} \, d3 = 2 \left[ \frac{3^2}{2} \right]_0^3 = 2 \left[ \frac{27}{2} \right]_0^3 = 2 \times \frac{27}{2} = 27 \]

\text{p. 279, 10 \(c\)}

\[ Z = \min(X, Y) \]

\[ P(Z \leq 3) = P(\min(X, Y) \leq 3) = \quad 1 - P(\min(X, Y) > 3) = \quad 1 - P(X > 3) \cap Y > 3) = \quad 1 - P(X > 3) P(Y > 3) = \quad 1 - (1 - P(X \leq 3)) P(1 - P(Y \leq 3)) \]
\[ F_2(3) = \begin{cases} 0 & ; 3 < 0 \\ 1-(1-3)^2 & ; 0 \leq 3 \leq 1 \\ 1 & ; 3 > 1 \end{cases} \]

\[ f_z(3) = \begin{cases} 0 & ; 3 < 0 \\ 2(1-3) & ; 0 \leq 3 \leq 1 \\ 0 & ; 3 > 1 \end{cases} \]

\[ E[\min(x, y)] = \int 3f_2(x)dx = \int 2 \cdot 3 \cdot (1-3) dx = 2 \left[ \frac{3^3}{3} \right]_0 - 2 \left[ \frac{3^3}{3} \right]_1 = 1 - \frac{1}{3} = \frac{2}{3} \]

\[ d) \quad Z = x^2 + y^2 \]

\[ P(Z \leq 3) = P(x^2 + y^2 \leq 3) \]

\[ F_2(3) = \begin{cases} 0 & ; 3 < 0 \\ \frac{\pi}{3} & ; 0 \leq 3 \leq 1 \\ -\frac{dA}{dz} & ; 1 \leq 3 \leq 2 \\ 0 & ; 3 > 2 \end{cases} \]

\[ A = \int_{-\sqrt{3}x^2}^{1} \int_{\sqrt{3}-1}^{\sqrt{3}+1} dxdy \]

\[ A' = \int_{-\sqrt{3}x^2}^{1} \int_{\sqrt{3}-1}^{\sqrt{3}+1} dxdy \]

\[ x = \sqrt{3} \sin \theta \]
\[ y = \sqrt{3} \cos \theta \]

\[ A = \int_{0}^{\frac{\pi}{3}} \int_{-\sqrt{3}}^{\sqrt{3}} r^2 dr d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} \left[ \int_{-\sqrt{3}}^{\sqrt{3}} r^2 dr \right] d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} \left[ \frac{3r^3}{3} \right]_{-\sqrt{3}}^{\sqrt{3}} d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \left( \sqrt{3} - \left( -\sqrt{3} \right) \right)^2 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \left( \sqrt{3} \right)^2 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 \cdot 1 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]

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\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]

\[ A = \frac{1}{3} \int_{0}^{\frac{\pi}{3}} 3 \cdot 3 \cdot 3 \cdot 2 \cdot 1 d\theta \]
\[ E(x^2 + y^2) = \int 3 f_z(3) \, dz \]
\[ = \int_{1}^{\frac{1}{2}} 3 \, dz + \int_{\frac{1}{2}}^{1} 3 \, dz \]
\[ = \frac{1}{2} \left[ \frac{3}{2} \right]_0^1 + \frac{1}{3} \left[ \frac{3}{2} \right]_1^2 \]
\[ = \frac{1}{8} \pi - \frac{2}{3} \int 3 A' \, dz \]
\[ = \frac{7}{6} \]
\( y \sim \text{uni}[0,1] \)

\( f_x(x) = \int_{y=0}^{1} f_{x|y}(x|y) f_y(y) \, dy \)

\[
= x \int_{y=0}^{1} \frac{1}{y} \, dy \\
= x \left( \ln y \right)_{0}^{1} \\
= x \ln \frac{1}{x} \\
\]

\( f_{x|y}(x|y) = \frac{1}{y} \)

\( f_{x,y}(x,y) = f_{x|y}(x|y) f_y(y) \)

\[
= \frac{1}{y} \\
= \frac{1}{y} \\
\]

\( f_x(x) = \int_{y=x}^{1} f_{x,y}(x,y) \, dy 
\]

\[
= \int_{y=x}^{1} \frac{1}{y} \, dy \\
= (\ln y)_{x}^{1} \\
= \ln(\frac{1}{x}) \\
\]

\( f_y(y) = \int_{x=0}^{y} f_{x,y}(x,y) \, dx 
\]

\[
= \int_{x=0}^{y} \frac{1}{y} \, dx \\
= \left( \frac{x}{y} \right)_{0}^{y} \\
= \frac{y}{2} \\
\]

\[
0 \leq y \leq 1 \\
\]

\( f_{x|y}(x|y) = \frac{1}{2y} \)

\( f_{x,y}(x,y) = f_{x|y}(x|y) f_y(y) \)

\[
= \frac{1}{2y} \cdot \frac{1}{y} \\
= \frac{1}{2y^2} \\
\]

\( f_y(y) = \int_{x=0}^{y} f_{x,y}(x,y) \, dx 
\]

\[
= \int_{x=0}^{y} \frac{1}{2y} \, dx \\
= \left( \frac{x}{2y} \right)_{0}^{y} \\
= \frac{y}{2} \\
\]

\( P_y(y) = \frac{1}{2} P_{x,y}(x,y) \)

\[
= \frac{1}{2} \frac{1}{15} (x+y) \\
= \frac{1}{15} \left[ (y+0) + (1+y) + (2+y) \right] \\
= \frac{1}{15} (1+y) \\
\]

\[
P_{X|Y}(x|y) = \frac{P_{X,Y}(x,y)}{P_y(y)} = \frac{\frac{1}{15} (x+y)}{\frac{1}{5} (y+1)} = \frac{1}{3} \frac{(x+y)}{(y+1)} \quad ; \quad x = 0, 1, 2 \]

\[
P(x=0|y=2) = \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9} \\
\]

\( p(x=0|y=2) \)